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NONLINEAR DYNAMICS UNDER UNCOVERED INTEREST RATE PARITY: CASES OF THE CZECH REPUBLIC, HUNGARY AND SLOVAKIA [1]

Abstract: *There has been an increasing amount of research giving mixed evidence of exchange rates following purchasing power and uncovered interest rate parities conditions. Using linear methods, a stable relationship between exchange rates and the respective fundamentals is usually found; however, the speed of adjustment of the exchange rates toward equilibrium seems to be rather slow. This together with the high volatility of both nominal and real exchange rates is addressed as one of the exchange rate puzzles. One approach to solving this puzzle is the use of nonlinear methods for exchange rate modelling. In this paper nonlinearity of the behaviour of the exchange rate of Czech Koruna, former Slovak Koruna and Hungarian Forint to Euro is tested using the ESTAR modelling framework. While it is usually the purchasing power parity condition which is tested, it is the uncovered interest rate parity which is considered in this paper.*

Key words: *ESTAR, nonlinear dynamics, uncovered interest rate parity*

JEL: C 22, E 44, F 31

Linear relationships between exchange rates and various fundamental variables have long been tested. The use of co-integration has been frequently applied to purchasing power parity (PPP), both uncovered and covered interest rate parities (UIP), monetary models of exchange rates, etc. Usually cointegrating vectors are found among the variables, indicating the presence of long-run relationships. Analyses of the adjustment process of the exchange rate towards the detected long-run relationship using the vector error correction model framework have shown that the adjustment of the exchange rate is slow. The half-life of the adjustment process has been estimated even up to 5 years in some papers. However, high volatility of both nominal and real exchange rates, which is well documented in the literature, indicates that the adjustment should be much faster, especially, if one takes account of the fact that it usually is nominal shocks to the exchange rate that are considered.

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The slow estimated adjustment of the exchange rates together with the high volatility of exchange rates under the conditions of nominal shocks represent one of the exchange rate puzzles, Rogoff [7] or Taylor and Taylor [11].

Both theoretical and empirical literature took up two basic approaches to the exchange rate puzzle. The first one relies on other than nominal fundamentals, i.e. real shocks to the exchange rate. One of the problems with this approach is that while it takes account of factors which do have effect on exchange rate behavior, it is hard to imagine that the occurrence of changes of real factors could be sufficient enough to explain the high variability of the exchange rates. The second basic approach to the puzzle is to discard the methods testing linear adjustment of the exchange rates as in fact the adjustment may be highly nonlinear in nature.

Of course, a relevant question to ask is: why should the adjustment of the exchange rate be considered nonlinear?

First, it is important to include the notion of transaction costs into the analysis. For the commodity arbitrage to be effective, the profit resulting from differences in prices in different economies must be cleared of the transaction costs which make the arbitrage possible. Thus, taking account of transaction costs, not any discrepancy among prices in various economies can represent a genuine arbitrage opportunity. While the transaction costs are certainly higher in the commodity market, it is not possible to disregard transaction costs in the financial market. However, its role in the interest arbitrage would be much smaller.

Second, one has to take account of expectations. Various agents differ in the way they perceive the given arbitrage opportunity. The transaction costs introduced in the previous paragraph are not the same for any agent. Thus, if one imagines a band of transaction costs according to various agents who might eventually take part in the arbitrage, it takes time for the arbitrage to operate fully. Also, agents differ in the way they evaluate the risk connected with the profit opportunity. Prices and returns may of course change, and it may take time to benefit from the profit opportunity. The role of expectations leads to two effects: first, it widens the transaction costs band so that an „objective“ information on transaction costs connected with a particular trading for particular agents may in fact be not enough to estimate the effective transaction costs band. Second, together with various transaction costs for various agents it means that the adjustment process of the exchange rate toward its equilibrium should be viewed as a smooth one. Both discrete and smooth adjustment process have been used to assess the nonlinear adjustment of exchange rates.

Thus, the main idea of modelling the nonlinear dynamics of exchange rates is that when the deviations of the exchange rate from the equilibrium (as given by the various fundamentals considered in the analysis) are relatively small, the exchange rate may, in fact, follow a random walk and does not need to exhibit an adjustment toward the equilibrium. The adjustment takes place when the deviations are significant. Therefore, when using linear methods to detect exchange rates adjustment toward equilibrium, one can draw a false conclusion that the adjustment is slow and, thus,

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defying theory. In fact, the adjustment of the exchange rate may be found to be in line with theoretical reasoning when nonlinear dynamics is put to use.

The modelling of discrete adjustment rests on using the TAR (threshold autoregressive) model. Smooth transition process of the exchange rate is on the other hand modelled by the STAR (smooth threshold autoregressive) models. With respect to the reasoning above, the STAR approach is taken in this paper.

There is a wide range of the STAR models depending on the particular form of the transition function. When the transition function takes on the form of exponential function, the resulting model is referred to as ESTAR, when the transition function is a logistic one, one refers to it as LSTAR, Terasvirta (1994). Other more sophisticated variants of the STAR models are also used, for example, Anderson, Vahid (2001) make use of USTAR model to capture the behaviour of the selected stock prices.

As far as smooth transition autoregressive models are concerned, ESTAR model is usually used to test the nonlinear adjustment of the exchange rates. LSTAR is not preferred because it suggests different speed of adjustment for a given absolute-value deviation of the exchange rate from equilibrium depending on whether the deviation is positive or negative.

The ESTAR model has been used to test the purchasing power parity, for example, by Panos et al [6], Taylor, Peel [9] or Taylor et al [10]. An attempt at verifying Balassa-Samuelson effect using ESTAR model can be found in Sager [8] while Liew et al [4] use it to test the monetary model of exchange rate.

In all of the studies mentioned above which use the ESTAR model, the model is estimated straightforward together with the transition parameter (see the Appendix) using a set of initial values. In this paper different approach is taken: the paper presents a discussion on this modelling framework, estimating a wide range of variants of the model according to the varying key parameter of the model and assessing the variants from the point of view of their stability and applicability to modelling the exchange rate deviations from the uncovered interest rate parity condition.

The methodology of the ESTAR modelling as used in this paper is presented in the Appendix. The main body of the paper presents the ESTAR model and the analyses of the model and examines their applicability. The main body of the paper is divided into four parts: the first part presents the key idea behind ESTAR model, the second part briefly presents the data used in the analyses, the results of the analyses are reported in the third part and the fourth part concludes.

1 ESTAR Model

Generally a STAR model may be presented as follows:

$$y_t = \sum_{j=1}^{p} \psi_j y_{t-j} + \left[\sum_{j=1}^{p} \psi_j^* y_{t-j} \right] \phi[\theta; y_{t-d}] + \varepsilon_t, \tag{1}$$

where y is a stationary process, ψ and ψ* are autoregressive parameters, ε is the disturbance term (independent and identically distributed, iid) and $\phi[\theta; y_{t-d}]$ is the transition function.

The transition function may take on various forms, here the exponential transition function is assumed:

$$\phi[\theta; y_{t-d}] = 1 - \exp\left[-\theta(y_{t-d})^2\right], \tag{2}$$

where θ is a positive parameter which determines the weights of the particular regimes.

Suppose θ approaches zero:

$$\lim_{\theta \to 0}\left[1 - \exp\left(-\theta(y_{t-d})^2\right)\right] = 0,$$

thus the transition function (2) takes on value zero. Which means that the STAR model becomes a simple linear autoregressive model:

$$y_t = \sum_{j=1}^{p} \psi_j y_{t-j} + \varepsilon_t. \tag{3}$$

The other extreme is when θ approaches infinity:

$$\lim_{\theta \to \infty}\left[1 - \exp\left(-\theta(y_{t-d})^2\right)\right] = 1,$$

then the transition function assumes value one and the STAR model becomes:

$$y_t = \sum_{j=1}^{p} \left(\psi_j + \psi_j^*\right) y_{t-j} + \varepsilon_t. \tag{4}$$

It is obvious that the exponential transition function is bounded in the interval. There are a few questions connected with the use of the ESTAR model.

The first one is the value of θ parameter. When the estimated value of this parameter is sufficiently low regarding the values of *y*, in fact there is hardly any regime switching in the data. There is a question whether this is a priori a problem. Because assuming the model is convenient for the data and the problem considered, it can just point to a slow mean-reversion or no mean-reversion at all even though nonlinear dynamics is used. However, it may also appear that the model is not suitable for the particular case.

The second question is the interdependancy between the parameter θ and ψ*. An inappropriate restriction on ψ* may lead to an unsound estimation of θ. Therefore, it seems a more reasonable to adopt a procedure which estimates the model using restriction on the parameter θ. In fact, a whole range of variants of the model may be estimated for several values of θ. This approach will be taken in this paper as opposed to the usual application of the model referred to above.

The third problem arising when using ESTAR model is its identification. It can be shown, Buncic (2007), that for small values of θ and/or small values of y, the model is hardly identified with respect to θ and ψ*.

Regarding these issues, the model will neither be estimated unrestricted nor with restrictions on the parameter ψ*. Rather a discussion of the model outcome with respect to various values of parameter θ will be presented. This enables to assess whether or not the model is appropriate for modelling exchange rate deviations with respect to the equilibria as given by UIP (uncovered interest rate parity) condition and if it is, what are the characteristics of the nonlinear adjustment of the exchange rate.

2 Data

Nonlinear dynamics was tested with respect to uncovered interest rate conditions. Data for the exchange rates (ER, amount of Czech Koruna per euro) and one-month money market interest rates (IR) were taken from Eurostat database. The sample period starts in January 1998 and ends in December 2008.

ADF test (see appendix) was used. Linear trend was never included and constant was included where suitable. The lags were chosen so as to minimize the SIC (Schwarz information criterion). The ADF tests presented in Table 1 show that the series may be considered I(1).

Table 1

ADF Tests

Variable	**t–statistic**	**Variable**	**t–statistic**
logarithmic exchange rate yoy changes		**logarithmic exchange rate yoy changes**	
$ER_{CZK/EUR}$	–2,557808	$ER_{CZK/EUR}$	–9,385774***
$ER_{HUF/EUR}$	–3,3142111*	$ER_{HUF/EUR}$	–7,994694***
$ER_{SKK/EUR}$	–2,129554	$ER_{SKK/EUR}$	–5,963353***
logarithmic exchange rate yoy changes		**logarithmic exchange rate yoy changes**	
$IR_{CZK}–IR_{EUR}$	–2,109895	$IR_{CZK}–IR_{EUR}$	–5,120503***
$IR_{HUF}–IR_{EUR}$	–2,403433	$IR_{HUF}–IR_{EUR}$	–7,767641***
$IR_{SKK}–IR_{EUR}$	–2,340347	$IR_{SKK}–IR_{EUR}$	–13,46870***

Annotation: *LHS gives results for levels, RHS gives results for 1st differences. (*** shows rejection of the null hypothesis at 1% level of significance).*

3 Results

First results for the estimation procedure laid out in the Appendix will be given and next the estimated models will be discussed.

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a) Building the ESTAR models

Following the procedure set up in the Appendix, as a starting point, regressions for UIP condition were run:

$$\Delta er_{D/F} = c + \beta_1 (IR_D - IR_F) + \varepsilon_t, \tag{5}$$

where er is log (ER), IR is interest rate, D and F means domestic and foreign, respectively (foreign for EU).

Interest rates were divided by 100 and the year-on-year change in exchange rate was computed as a logarithmic change.

The implication of UIP is straighforward: coefficient β_1 should be positive, i.e. the increase in the interest rate differential should lead to domestic currency depreciation. Assuming the expected spot exchange rate is higher than the current spot exchange rate (the domestic currency is expected to depreciate) then in the short run a rise in domestic interest rate tends to lower (appreciate) the exchange rate as the demand for domestic assets rises. However, the spot appreciation given the expected spot exchange rate means higher future depreciation of domestic currency. This is the long-run implication of the UIP condition. Table 2 reports the results for regression (5).

Table 2

Regression Tests

CZ			HU			SK		
c	–0,00344	–8,773546***	**c**	–0,0028	–2,790643***	**c**	–0,00587	–15,04721***
β_2	0,09067	3,461731***	**β_2**	0,04858	3,426208***	**β_2**	0,12065	14,63427***
adj R	0,085155		**adj R**	0,083416		**adj R**	0,643679	
AIC	–8,124493		**AIC**	–8,042988		**AIC**	–8,730038	
DW	0,168526		**DW**	0,199832		**DW**	0,360068	
JB	0,410479		**JB**	0,43071		**JB**	0,792574	

Annotation: *(*** , ** shows rejection of the null hypothesis at 1% level of significance and 5% level of significance, respectively).*

The coefficients have the anticipated signs. However, the regressions are spurious as the variables exhibit unit root behaviour. Also one can notice the residuals are highly autocorrelated as the Durbin-Watson statistic (DW) reaches very low levels in both cases. According to Jarque-Bera statistic (JB) normality of residuals cannot be discarded for any case.

Residuals from regression (5) were tested for unit root behaviour. First ADF test was run. In all the three cases the test rejects unit root behaviour at 1% level of significance. However, as it is noted in the Appendix, ADF may not be appropriate when nonlinearity of residuals is assumed. Thus, the KSS test was also run. Table 3 presents the results for both tests. In the case of KSS test t-statistics for the significance of coefficient ρ are given. The null hypothesis of ρ being equal to zero

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is rejected at 1% level of significance in all the three cases. The residuals are thus considered stationary and the long-run relationships (co-integrating vectors) among the respective variables are confirmed.

Table 3

Tests of Stationarity of Residuals

ADF test		KSS test	
CZ		CZ	
t-statistic	–2,697307***	t–statistic	–3,367875***
HU		HU	
t-statistic	–3,580590***	t–statistic	–2,535237**
SK		SK	
t-statistic	–4,421389***	t–statistic	–4,120886***

Annotation: *(*** shows rejection of the null hypothesis at 1% level of significance).*

Following the outline of the analysis given in the Appendix, linearity tests were performed on the residuals.

To run the linearity test, the order of autoregression needs to be decided. The order was chosen by inspection of the partial autocorrelation functions (PACF) for the respective residuals. The ACFs and PACFs for lags up to 10 are given in Table 4.

Table 4

ACF and PACF of Residuals

	CZ		HU		SK	
lag	ACF	PACF	ACF	PACF	ACF	PACF
1	0,902	0,902	0,892	0,892	0,818	0,818
2	0,793	–0,109	0,727	–0,233	0,569	–0,190
3	0,670	–0,133	0,578	0,057	0,414	0,180
4	0,550	–0,049	0,437	–0,116	0,280	–0,173
5	0,450	0,032	0,312	–0,003	0,125	–0,103
6	0,349	–0,083	0,193	–0,110	0,008	0,009
7	0,252	–0,069	0,067	–0,128	–0,080	–0,112
8	0,164	–0,026	–0,046	–0,030	–0,171	–0,088
9	0,072	–0,100	–0,143	–0,072	–0,257	–0,091
10	0,008	0,071	–0,217	–0,010	–0,304	–0,027

Significant partial autocorrelation is obvious at lag 1 for all three economies. Therefore the order of autoregression was set to 1. The goal of the auxiliary regression is to decide whether or not there is nonlinearity in the residuals and if there is, what the order of delay (d) would be, i.e. at which lag the nonlinearity

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is most significant. Therefore the regression was run starting with d equal 1 and continuing up to d equal 12.

The regressions themselves are not presented as they carry little information. Rewriting the auxiliary regression given in the Appendix for p = 1:

$$y_t = \alpha_{00} + \alpha_{01} y_{t-1} + \alpha_{11} y_{t-1} y_{t-d} + \alpha_{21} y_{t-1} y_{t-d}^2 + \varepsilon_t, \qquad (6)$$

the Wald coefficient restriction test was performed at suspicious delays in the form:

$$\alpha_{11} = \alpha_{21} = 0.$$

In the cases of the Czech Republic and Slovakia the strongest nonlinearity in the residuals was discovered at the delay of order 6; for Hungary the delay is 2. The results for the Wald coefficient restriction test are given in Table 5.

Table 5

Wald Restriction Tests within Tests of Linearity

CZ		HU		SK	
lag	6	lag	2	lag	6
F-statistic	3,188829**	F-statistic	4,029089**	F-statistic	5,325492***

Annotation: *(** shows rejection of the null hypothesis at 5% level of significance).*

In the cases of the Czech Republic and Hungary the null hypothesis is rejected at 5% level of significance; in the case of Slovakia it is rejected at 1% level of significance. Next step is setting up the ESTAR models.

For the purpose of clarity, the ESTAR model (1) is rewritten given the parameters estimated in the previous steps. While the order of autoregression is the same for the three economies and delay parameter is different for Hungary on the one hand and the Czech Republic and Slovakia on the other:

$$CZ, SK: y_{UIPt} = \psi_1 y_{UIPt-1} + \left[\psi_1^* y_{UIPt-1}\right]\left[1 - \exp(\theta y_{UIPt-6}^2)\right] + \varepsilon_t, \qquad (1a)$$

$$HU: y_{UIPt} = \psi_1 y_{UIPt-1} + \left[\psi_1^* y_{UIPt-1}\right]\left[1 - \exp(\theta y_{UIPt-2}^2)\right] + \varepsilon_t. \qquad (1b)$$

Instead of estimating the ESTAR models unrestricted and giving the results, estimation for various θs for both cases is presented.

After inspection of the behaviour of the models with respect to varying θs, the range of possible θs was set for each case. In this first step the criterion was such that the estimated autoregressive parameters must be statistically significant at 5% level at least.

Under such conditions the possible interval of θs for the Czech Republic is ⟨10 000, 700 000⟩, for Hungary it is ⟨5 000, 360 000⟩ and for Slovakia it is ⟨4 000, 110 000⟩. The estimated models for the border values of θs and for two other additional values of the parameter are given in Table 6.

Table 6

ESTAR Models

CZ			**HU**			**SK**		
(θ=10000)			**(θ=5000)**			**(θ=4000)**		
ψ	1,013696	19,77912***	**ψ**	1,016923	15,26619***	**ψ**	0,956892	14,97048***
ψ*	–0,460971	–2,201066**	**ψ***	–0,751722	–2,309178**	**ψ***	–2,377798	–2,712463***
adj R	0,868204		**adj R**	0,813234		**adj R**	0,731294	
AIC	–10,14695		**AIC**	–9,738929		**AIC**	–10,10890	
BG	2,040484*		**BG**	5,490696**		**BG**	3,914861**	
ARCH	0,565495		**ARCH**	1,288268		**ARCH**	1,51634	
JB	3,838053		**JB**	2,473009		**JB**	1,988448	
(θ=100000)			**(θ=100000)**			**(θ=50000)**		
ψ	1,093274	16,35665***	**ψ**	1,263317	9,595683***	**ψ**	0,993435	13,05876***
ψ*	–0,259787	–2,834533***	**ψ***	–0,448388	–2,941442***	**ψ***	–0,427228	–2,550341**
adj R	0,871736		**adj R**	0,818248		**adj R**	0,729343	
AIC	–10,17412		**AIC**	–9,766144		**AIC**	–10,10166	
BG	1,757064		**BG**	3,974039**		**BG**	4,011307**	
ARCH	0,618963		**ARCH**	1,887584		**ARCH**	1,786405	
JB	4,807459*		**JB**	2,537306		**JB**	2,120737	
(θ=200000)			**(θ=300000)**			**(θ=100000)**		
ψ	1,118924	14,29553***	**ψ**	1,282143	6,801978***	**ψ**	0,997882	11,23817***
ψ*	–0,260440	–2,680478***	**ψ***	–0,418311	–2,108426**	**ψ***	–0,312960	–2,072618**
adj R	0,870814		**adj R**	0,811847		**adj R**	0,724159	
AIC	–10,16695		**AIC**	–9,731532		**AIC**	–10,08269	
BG	1,811954		**BG**	5,145422**		**BG**	3,808371**	
ARCH	0,634063		**ARCH**	2,318689		**ARCH**	2,218916**	
JB	4,066182		**JB**	2,084957		**JB**	2,472981	
(θ=700000)			**(θ=360000)**			**(θ=110000)**		
ψ	1,135624	10,36243***	**ψ**	1,279439	6,458465***	**ψ**	0,996226	10,94070***
ψ*	–0,241094	–1,977273**	**ψ***	–0,411589	–1,987934**	**ψ***	–0,297471	–1,978817**
adj R	0,867132		**adj R**	0,811066		**adj R**	0,723246	
AIC	–10,13885		**AIC**	–9,727391		**AIC**	–10,07939	
BG	2,074182*		**BG**	5,405768***		**BG**	3,766798**	
ARCH	0,695183		**ARCH**	2,254121		**ARCH**	2,302178**	
JB	2,679557		**JB**	2,044592		**JB**	2,556444	

Annotation: *(***, **, * shows rejection of the null hypothesis at 1%, 5%, 10% level of significance respectively).*

The models for the Czech Republic explain around 87% of variability of the deviations. The residuals of the ESTAR model are relatively well behaved, showing no remaining heteroskedasticity (White test, ARCH, null of no remaining heteroskedasticity), little or no serial correlation (Breush-Godfrey, BG, null of no serial correlation), and may be considered normal (Jarque-Bera, JB, null of normal distribution).

The models for Hungary explain around 81% of variability of the deviations. The residuals show no remaining heteroskedasticity and follow normal distribution, however, they show some serial correlation.

The models for Slovakia explain around 73% of variability of the deviations. The residuals follow normal distribution; however, they show some serial correlation and in some cases remaining heteroskedasticity. As well as in the case of Hungary higher values of θ worsen the model's diagnostics.

The residuals from the ESTAR models were further tested for remaining nonlinearity. The tests showed no statistically significant nonlinearity.

b) Assessing the ESTAR Models

There are two important questions to ask to assess the stability and applicability of the ESTAR model for modelling the deviations from UIP condition. The first question is to what extent the two extreme regimes of the adjustment process as shown in (3) and (4) are used when considering the values of the residuals and varying θs, i.e. the key parameter of the transition function and whether it reflects economic reasoning. In other words this means assessing the value of the transition function with respect to varying θs for given magnitudes of the deviations from the equilibrium. If the value of the transition function is one or close to it, then there should be, other things equal, strong autoregression pulling the exchange rate towards the equilibrium value given by the UIP condition. In the other extreme, the value of transition function being 0 or close it, means the exchange rate follows the random walk or even an explosive process.

The second important question is closely connected with the first one. If for given residuals and value of θ, i.e. for given value of transition function, the process should be such as given by (4) or similar to it, is it sure that the process will be stable? If one considers the estimated values for the autoregressive coefficients in Table 6, it is obvious that the coefficient ψ is always positive and very often greater than 1 while the second coefficient ψ* is always negative and usually small in absolute value. Generally, if the pure random walk is ruled out, i.e. the value of transition function is not 0, the autoregression is given by the following equation whose extreme version is equation (4) for the value of transition function 1:

$$y_t = \left(\psi + \varphi\psi^*\right) y_{t-1} + \varepsilon_t \,, \qquad (7)$$

where φ is the value of the transition function.

If the process is to be stationary (mean-reverting), the following must hold: $|\psi + \varphi\psi^*| < 1$. From (7) one can easily see that given the values of the estimated autoregressive coefficients ψ and ψ*, the process as a whole can be as well as explosive for some values of transition function φ. The question is for what range of the values of the transition function this happens, and if it is in line with economic reasoning.

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The deviations were in all three cases taken from the interval ⟨–0,005, 0,005⟩, which means around 75% of deviations for the Czech Republic and Hungary and around 85% of deviations for Slovakia.

The intervals for θs for each countries were for the purpose of the following analyses divided as follows: for the Czech Republic ⟨10 000, 10 000 + *k*,..., 700 000⟩, where k = 10 000, for Hungary ⟨5 000, 5 000 + *k*,..., 360 000⟩, where k = 5 000 and for Slovakia ⟨4 000, 4 000 + *k*,..., 110 000⟩, where k = 2 000.

First the values of transition function for values of the deviations: 0,001; 0,002; 0,003; 0,004 and 0,005 and for the given intervals of θs were computed. The values of the transition function is, regarding equation (2), the same for the negative counterparts of the deviations. The evolution of the value of transition function is given in Figures 1 – 3 below.

Figure 1

The Sensitivity of Transition Function on θ Given Values for Residuals. CZ

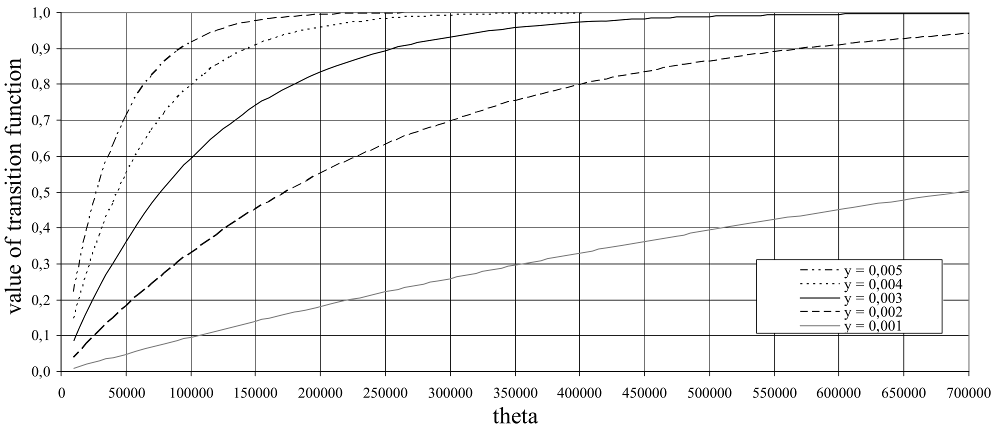

Figure 2

The Sensitivity of Transition Function on θ Given Values for the Residuals. HU

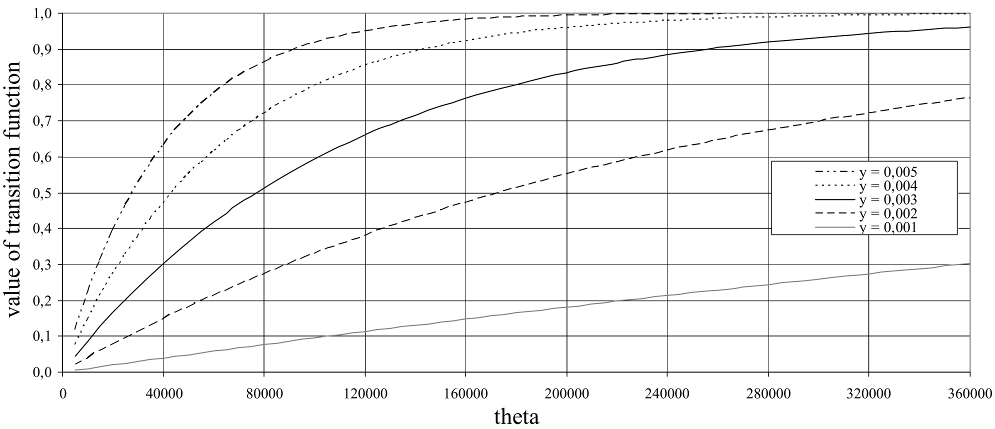

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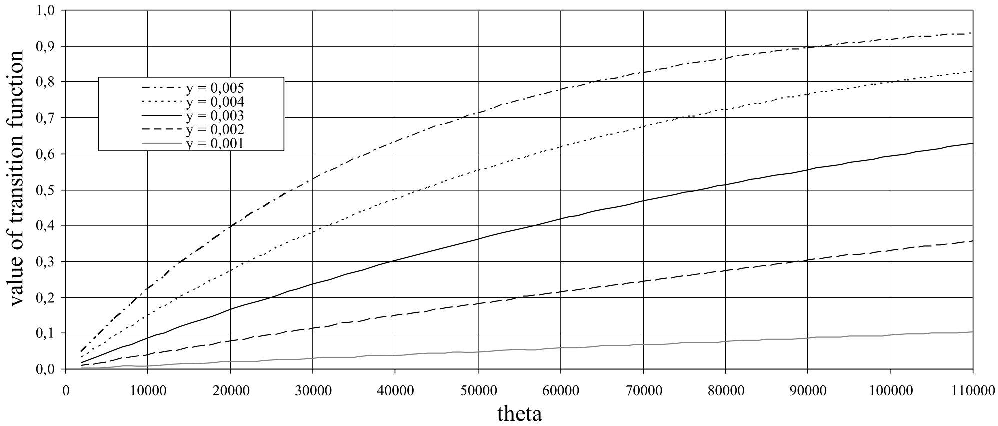
Figure 3

The Sensitivity of Transition Function on θ Given Values for Residuals. SK



In the case of the Czech Republic the value of transition function gets very close to 1 for residuals above 0,003 and around the middle of the interval for θs. This means that there is a clear tendency for the outer regime (transition function equal 1) to come into play. Thus for large deviations there is a potential for a stationary autoregression process. Still taking account of (7), it needs to be verified that it really is the case.

In the case of Hungary the value of the transition function nears 1 for residuals above 0,004 just beyond the middle of the range for θs.

In the case of Slovakia the results give somewhat different picture. Only for the largest residuals and values of θs does the value of transition function nears 1. Thus the estimate for Slovakia shows the least tendency of the exchange rate to return to the equilibrium as given by UIP.

Figure 4

The Value of Transition Function for Given Range of Residuals and Given θs. CZ

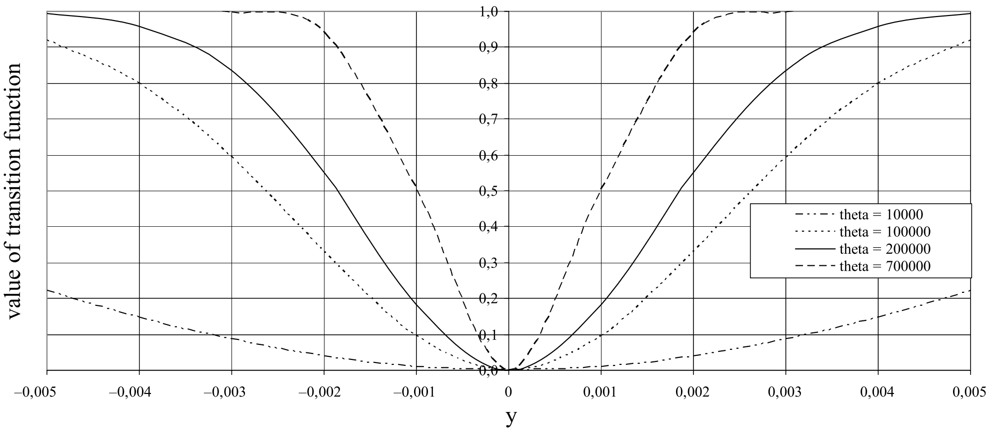


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Figure 5

The Value of Transition Function for Given Range of Residuals and Given θs. HU

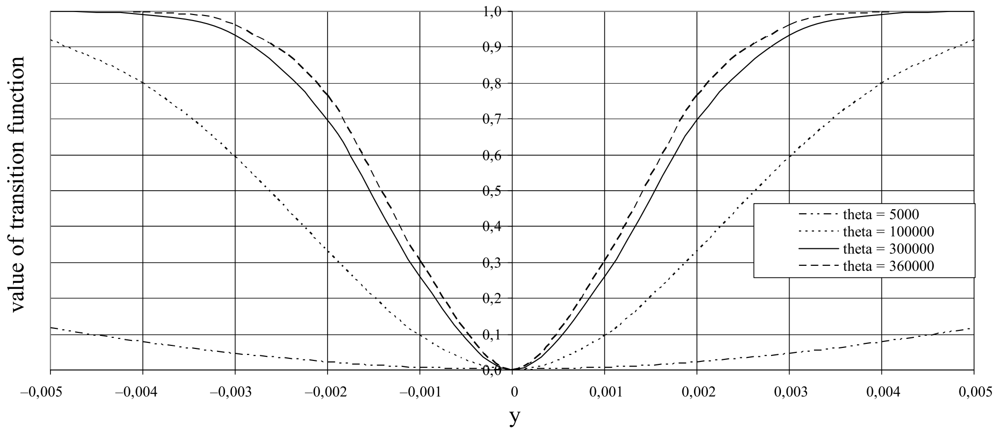
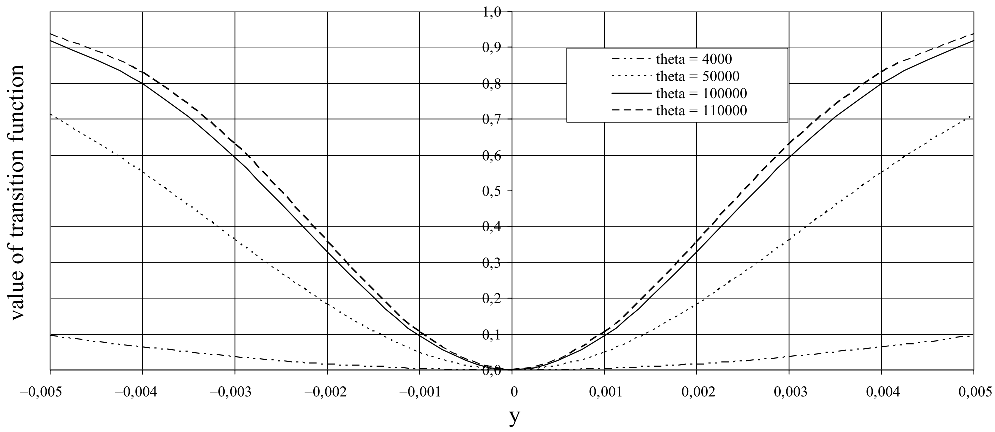


Figure 6

The Value of Transition Function for Given Range of Residuals and Given θs. SK



One important conclusion from this part of the analysis is that for higher values of deviations, and given range of θs the value of transition function is much more stable for the Czech Republic and Hungary than for Slovakia.

For better understanding of the impact of the magnitudes of residuals and θs on the value function, figures 4–6 depicts the value functions for the whole range of residuals and selected values of θs, those used in the examples of the estimated ESTARS in Table 6.

Generally, one can see that with increasing θ, the value of transition function increases for any value of residuals (except 0). Again Figure 6 shows that in the case of Slovakia even for the largest possible θ (so that the estimated parameters are significant at least 5% level), and largest deviations (y), the value of transition function just climbs over 0,9.

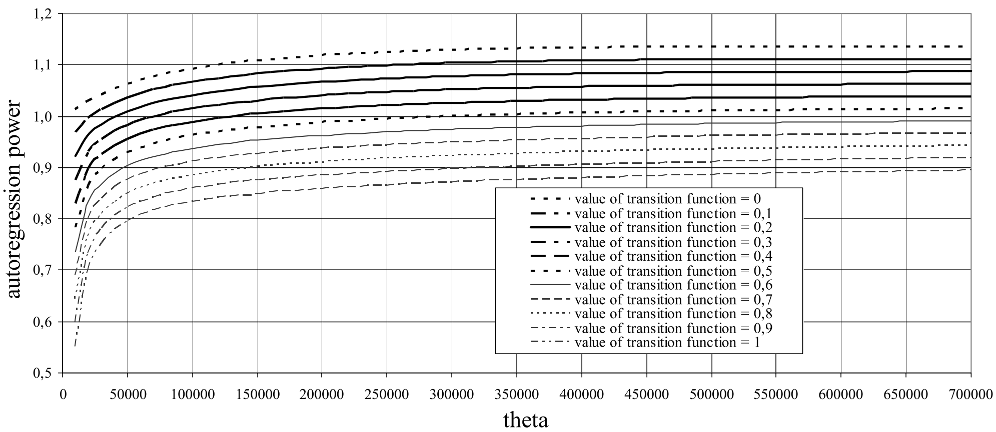
Based on this analysis, it can be concluded that the model seems suitable for the cases of the Czech Republic and Hungary; however, it may not be so in the case of Slovakia. There are three reasons why in the case of Slovakia the tendency of the exchange rate toward the equilibrium may be the lowest from the three cases: first, the behaviour of the exchange rate may be influenced by the preparations of Slovakia for EMU entry, second, the model is not suitable for the dynamics of Slovak Koruna (the nonlinear dynamics may be different in nature from the one this model is able to capture) and third, combination of both.

Before reaching definite conclusions, it is, however, necessary to check the stability of the autoregression process as it is implied by (7).

This was done by estimating the ESTAR models for the whole range of θs given by the intervals for the three economies considered in the analysis and taking the estimated autoregressive coefficients ψ and ψ* and computing the total autoregression power as given in (7) for the range of values of transition function: 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9 and 1,0. The results for each economy are depicted in figures 7–9.

Figure 7

Stability of the Dynamics for Given Range of θs and Values of Transition Function. CZ



There are three facts to bear in mind when examining the figures. First, if the total autoregression power exceeds 1 in absolute value for given value of transition function and some values of θ, then it is depicted in bold. Second, one should keep in mind the fact that generally higher value of transition function means higher deviations of the exchange rate from the equilibrium given θ (see Figures 4 – 6). Third, the order of the functions for given value of transition function copies the order given in the legend.

In the case of the Czech Republic the autoregressive process may be explosive for values of transition function up to 0,5. Taking account of the previous analysis it means that it can be explosive for relatively low values of residuals when θ is relatively higher which is in line with economic theory or for the whole range of the

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residuals when θ is relatively lower. Thus this puts another constraint on the range of θs. Considering the set of residuals for which there should be a tendency of the exchange rate to move towards the equilibrium to be 〈–0,005, –0,003〉 ∪ 〈0,003, 0,005〉, then θ should be from the interval 〈80 000, 700 000〉.

Figure 8

Stability of Dynamics for Given Range of θs and Values of Transition Function. HU

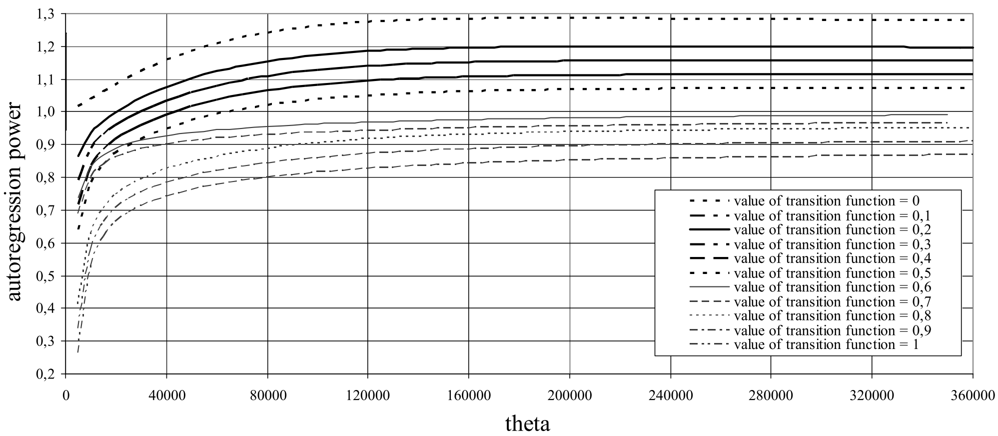
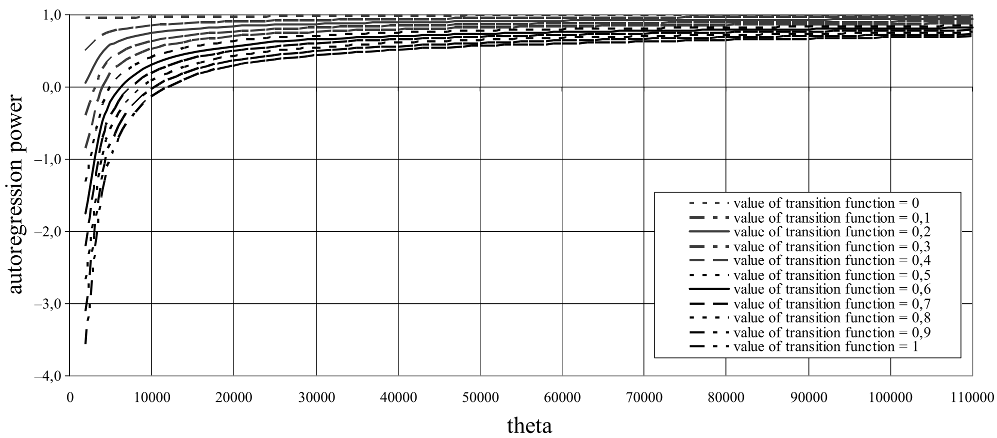


Figure 9

Stability of Dynamics for Given Range of θs and Values of Transition Function. SK



In the case of Hungary the autoregressive process may be explosive for values of transition function up to 0,5, which is the same as in the case of the Czech Republic. Again considering the same set of residuals for which the exchange rate should follow a stationary process, i.e. 〈–0,005, –0,003〉 ∪ 〈0,003, 0,005〉, θ should be from the interval 〈80 000, 360 000〉.

The picture for Slovakia is quite different. The values of transition function for which the exchange rate may follow an explosive process are 0,5 and above. Apart form the previous cases, there is never the case of the total autoregression power (coefficient) exceeding the value 1, but rather being negative and lower than –1 for relatively low values of θs. To eliminate this problem θ must be higher than 6 000, i.e. from the interval ⟨8 000, 110 000⟩ when regarding the step considered – k = 2000.

High values of estimated transition parameter θ can be found in Panos, Nobay, Peel [6] for some of the cases they examined in their analysis.

In the cases of Hungary and Slovakia it is important to bear in mind that highest values of θ worsen the model's diagnostic, especially, increasing the intensity of serial correlation among residuals and causing some heteroskedasticity in the case of Slovakia. In effect the intervals should be cut shorter on the right side.

Finally, using the estimated total autoregression power, one can obtain an idea of the speed of adjustment of the exchange rate towards the equilibrium. The deviations above 0,003 and the intervals of θs are considered above.

For the Czech Republic, the exchange rate starts to react to the shock with a six-month delay. Then regarding the values of the autoregression power, the half life of the shock after the exchange rate starts to react is less than a month. Thus it can be inferred that the shock is absorbed after less than 8 months.

For the case of Hungary the half life of shock is more or less the same; however, in this case the exchange rate reacts with a delay of two months. Altogether this means that the shock is absorbed in less than 4 months.

In the case of Slovakia the half life of the shock is much more variable regarding the exact magnitude of the deviation (still 0,003 and above) and parameter θ. The half life can vary from approximately a month to two and half months. Taking account of the fact that the exchange rate starts to react with a delay of six months, the shock may not be absorbed in less than 10 months.

Especially in the case of Slovakia one should take account of the fact that high values of θ bring increasing serial correlation among residuals and some heteroskedasticity into the ESTAR models. In effect the shock should be considered to be absorbed in more than 10 months but less than a year.

Durčáková, Mandel and Tomšík (2005) using the co-integration and error correction methods estimated the adjustment parameters for the Czech Republic, Hungary at app. –0,34 and –0,42, respectively. However, their analysis includes other factors besides interest rate differential such as ratio of domestic and foreign price levels, FDI or foreign debt. In addition to this, interest rate differential does not enter in the co-integrating vector in the case of the Czech Republic and is used only as an exogenous variable in the error correction model. They found no co-integration relationship for Slovakia. Their analysis was performed on a monthly basis. This means that the shocks to the exchange rate may be absorbed in 4 and 3 months, respectively. Regarding the results of their analysis, it may be concluded that if the sole factor examined had been the interest rate differential, the shocks to the exchange rate would have been much more resistant.

4 Conclusions

In the paper the ESTAR models were developed to describe and analyze the adjustment of the exchange rate of Czech Koruna, Hungarian Forint and the former Slovak Koruna to Euro with respect to the uncovered interest rate condition.

The models were not estimated unrestricted as the ESTAR models have problem with identification when modelling exchange rates. The restriction was placed on the value of the parameter of the transition function and a whole range of models were analyzed via varying values of the transition function parameter.

The ESTAR model showed that for relatively smaller values of deviations (smaller values of the transition functions) or smaller values of the transition parameter (smaller values of the transition functions) the process of "adjustment" displays even explosive or unit root behaviour, which is in line with economic reasoning. The process becomes stationary and more intensive with high deviations of the exchange rate from the equilibrium or high values of the transition parameter.

The estimated models for Slovakia showed larger persistence in the behaviour of the exchange rate which may be caused by the fact that in the last years of the sample Slovakia was preparing for the EMU entry and, of course, the model may not be suitable to capture the dynamics in this case.

For reasonably high deviations the adjustment process is mean reverting and the estimated time for a shock to sound off reaches up to 8 and 4 months for the Czech Republic and Hungary, respectively. In the case of Slovakia the shock need not be absorbed until 10 months for the deviations of the same magnitude. The residuals are well-behaved and do not show any remaining nonlinearity. The ESTAR framework seems to be suitable for the modelling of deviations of the exchange rate from the UIP in the cases of the Czech Republic and with the exception of otherwise acceptable high values of the transition parameter Hungary. The results for Slovakia are not so supportive; however, they may be influenced by the preparations of Slovakia for the EMU entry.

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Appendix

Estimation Procedure

First, it is important to establish whether a long-run relationship among the variables exists as it is the deviations from this relationship which is modelled.

If we assume a vector of I(1) variables, then a long-run relationship (cointegrating vector) exists when the disturbance term in the following regression is I(0):

$$x_t = \alpha_0 + \sum_{j=1}^{n} \alpha_j z_j + \varepsilon_t, \tag{1}$$

where x is a dependent variable (exchange rate) and z are independent variables (price levels, interest rates), ε is a disturbance term and α_0 is a constant.

After the deviations (residuals) are obtained, they can be tested for unit root behaviour. To test if the input series are integrated of the same order, one can use a wide range of unit root tests. Standard Augmented Dickey-Fuller (ADF) test is used in this paper:

$$\Delta y_t = \alpha y_{t-1} + x_t'\delta + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + ... + \beta_p \Delta y_{t-p} + \varepsilon_t, \tag{2}$$

where y is the examined series, x represents the exogenous factors (constant, linear trend or nothing). The null hypothesis is such that α = 0 which means the series exhibits unit root behaviour.

To check the series of residuals from equation (1), ADF is also used; however, as Liew et al (2008) point out, in the presence of nonlinearity the ADF may not be relevant. Therefore KSS test is used for the residuals as well:

$$\Delta y_t = \rho y_{t-1}^3 + \upsilon_t. \tag{3}$$

The null hypothesis is $\rho = 0$, which means that the series is nonlinear and nonstationary. If the alternative is accepted, nonlinear co-integrating relationships among the series exists. The hypothesis will be tested using t-statistics.

If the series are found to be co-integrated, the deviations may be tested for nonlinearity. This test also determines the delay parameter.

To check for nonlinearity an auxiliary regression is run, for example Luukkonen et al (1988):

$$y_t = \alpha_{00} + \sum_{j=1}^{p} \left(\alpha_{0j} y_{t-j} + \alpha_{1j} y_{t-j} y_{t-d} + \alpha_{2j} y_{t-j} y_{t-d}^2 \right) + \varepsilon_t, \qquad (4)$$

where y are the residuals from (1), α_{00} is constant and $\alpha_{0j,}$ $\alpha_{1j,}$ α_{2j} are estimated parameters and ε are residuals of the auxiliary regression (4).

The null hypothesis may be stated:

$$H_0 : \alpha_{1j} = \alpha_{2j} = 0 \;\; j = 1,..., p .$$

If the null holds that there is no nonlinearity in the residuals.

An important information necessary for the test of linearity is the order of autoregression *p*. The order of autoregression is judged according to PACF (partial autocorrelation function), Terasvirta (1994).

If the residuals from (1) are found to exhibit nonlinearity, they are plugged into the ESTAR model. As it was stated above, the ESTAR models will not be estimated unrestricted, rather a wide range of ESTAR models will be estimated according to varying parameter θ.

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