Ivan Brezina
Zuzana Čičková
Juraj Pekár

APPLICATION OF EVOLUTIONARY APPROACH TO SOLVING VEHICLE ROUTING PROBLEM WITH TIME WINDOWS[1]

Abstract: *This article deals with approach to solving the vehicle routing problem with time windows (VRPTW) that is based on evolutionary algorithms. Because the vehicle routing problem with time windows belongs to the group of NP-hard problems, the use of optimisation techniques seems to be relatively complicated; therefore nowadays many researchers turn their attention to the application of alternative computational techniques that are inspired by evolutionary biology. Besides the unconstrained problems with continuous variables, the evolutionary algorithms may be used also for dealing with constrained problems with discrete variables; however, their use requires the techniques that enable to decode a candidate solution into the problem solution so that all given criteria are met. The authors present their own approach to coding and decoding the VRPTW that enables to use a lot of evolutionary techniques.*

Keywords: *vehicle routing problem with time windows, evolutionary algorithms*

JEL: C 6, C 61, C 63

Introduction

Recently most discussed problems related to the depletion of non-renewable resources that are necessary for car propulsion, therefore they increase the cost (oil crisis, gas crisis), result in an increased interest in the development of instruments that enable the optimisation. The efficiency can be also increased by force of quantitative approaches that are aimed at optimization of physical distribution of the goods and distribution costs. Related optimization problems are routing and scheduling problems: shortest path problem, travelling salesman problem, vehicle routing problem, fixed schedule problem, pick-up and delivery problem, etc.

[1] This research was supported by the grant project of the Slovak Ministry of Education VEGA No. 1/4588/07 "Reverse Logistics Modelling – Optimisation of Recycling and Disposal Processes".

The management of physical distribution of goods is interesting not only for its practical applicability, but also for theoretical research, because this type of problems belong to the NP-hard problems, that can be characterized as problem, cannot be solved in polynomial time by deterministic algorithm, and therefore it is difficult to obtain optimal solution (usually for large-scale problems).

The practical problems of physical distribution often include the need to respect the time restriction. Oftentimes, we consider time restriction that are a consequence of earliest possible time of service, the latest possible time of service, the service during the given time interval etc. The above mentioned terms are known as time windows. If it is necessary to consider only the earliest possible time of service or the last possible time of service, the problem is known as problem with soft delivery time windows constraints, in case of restriction with time interval with given lower and upper limit the problem is known as problem with hard delivery time windows constraints, e.g. Desrosiers, Dumas, Solomon, and Soumis [21].

Further on, the most important problems with time windows will be presented. Even if those problems are well known, we try to introduce the sources that indicate the progress in their formulation and solution.

1 The most Important Problems

The routing and scheduling problems generally involve the assignment of vehicle (fleet of vehicles) to trips such the corresponding cost is minimal. The models often need to be generalized by time dimension in the form time window constraint in that way that by some supply nodes the earliest time of service of the node need to be during time interval, so that it must not begin before the pre-specified limit and must not finish after a pre-specified deadline, when each supply node needs a specified time to serve. A vehicle is not permitted to arrive at a supply node after the deadline. However, sometimes it is possible to wait for service when a vehicle arrives before the earliest limit (the models include waiting). Some of the models propose the restriction only in the way the vehicle needs to arrive within a specific time window, some modes involves breaking the time window at the cost (penalty). Sometimes it restricted also the earliest possible time in which the vehicle can leave the depot and (or) the latest possible time in which the vehicle can arrive to the depot. All of the problems can be formulated on a weighted graph, where the weights represent distance, time, cost, etc.

A) The shortest path problem with time windows

Problem: The problem appears as a sub-problem in many time-constrained problems. The goal is to minimize the total travelled physical or time distance in a graph between two specified nodes (one initial node and one final node) through the service of the transit nodes needed to start within a specified time interval (or not start before and not finish after a time window limit), when the weight (distance, time,

cost etc.) of the arc between the corresponding nodes is known. The mathematical programming formulation deals with two class of variables – binary flow variables (indicates if the arc between the nodes is used or not) and time variables (duration of the arc between two nodes, start of service of the node etc.) with the objective function that allows to minimize the total cost (distance, time etc.).
References: Above mentioned problem is presented in e.g. Joksch – 1966 [33], Minoux – 1975 [40], Hansen – 1980 [27], Aneja, Aggarwal and Nair – 1983 [2], Jaffe – 1984 [31], Psaraftis, Solomon, Magnati and Kim – 1990 [47], Tsitsiklis – 1992 [53], Desrosiers et al. – 1995 [21], Desaulniers and Villeneuve – 2000 [19], Donati, Montemanni, Gambardella and Rizzoli – 2003 [23], Qureshi, Taniguchi and Yamada – 2007 [48], Androutsopoulos and Zografos – 2008 [1] etc.

B) The travelling salesman problem with time windows

Problem: The goal is to minimize the total travelled physical or time distance travelled by a vehicle starting and returning at the same node (central node) that must visit a set of other nodes (supply nodes) within their time windows. Thus, the central node is initial node and also final node and all other nodes in a graph are transit nodes and the weight (distance, time, cost etc.) of the arc between all the nodes is known. Further on we consider that for every supply node the time window with identical meaning as in the shortest path problem is known. It is necessary to determine the sequence of nodes, in which the nodes will be visited, with respect to following conditions: the vehicle starts and finish at the initial node, needs to visit each supply node once and exactly once and the time window of each supply node will be met (we consider that the capacity of vehicle is unlimited). The mathematical programming formulation contains also binary flow variables (indicates if the arc between the nodes is used or not) and time variables (duration of the arc between two nodes, start of service of the node etc.) with the objective function that allows to minimize the total cost (distance, time, etc.).
References: Above mentioned problem is presented in e.g. Or – 1976 [43], Carpaneto and Toth – 1980 [10], Baker – 1983 [4], Savelsbergh – 1985 [50], Cyrus – 1988 [11], Desrochers, Desrosiers and Solomon – 1992 [20], Langevin, Desrochers, Desrosiers, Gelinas and Soumis – 1993 [35], Desrosiers, Dumas, Solomon and Soumis – 1995 21], Carlton and Barnes – 1996 [9], Gendreau et al. – 1999 [26], Ascheuer, Fischetti and Grőtschel – 1999 [3], Bar-Yehuda, Even and Shahar – 2003 [6], Fábry – 2006 [25], Zhang and Tang – 2007 [55] etc.

C) The vehicle routing problem with time windows

Problem: This problem is the generalization of travelling salesman problem, where we consider that the capacity of vehicle (fleet of vehicles) is limited and that the demands of nodes as well as their time windows are known. The demand need to be fulfilled from initial node (central node, depot). The goal is to minimize the total

travelled physical or time distance as a sequence of routes from depot to other nodes, with respect to following criteria: the central node is the initial node and also the final node of every route, the other nodes in a graph are transit nodes with certain demand, from the depot the demands of the other node within their time windows need to be met, each node (except central node) is visited exactly once and sum of all demands on route must not exceed the capacity of the vehicle. The mathematical programming formulation is involved under the one of following restrictions: the number of vehicle is free or the capacity at least one vehicle is unlimited (the objective often includes the determination of vehicle fleet size). The model also deals with binary and time class of variables.
References: Above mentioned problem is presented in e.g. Magnanti – 1981 [38], Bodin, Golden, Assa and Ball – 1981 [7], Desrosiers, Soumis and Desrochers – 1984 [22], Laporte and Nobert – 1987 [36], Potvin and Rousseau – 1993 [46], Homberger and Gehring – 2005 [29], Tan, Lee, Zhu and Ou – 2001 [52], Peško – 2002 [44], Janáček – 2003 [32], Lau, Sim and Teo – 2003 [37], Vacic and Sobh – 2004 [54], Bräysy and Gendreau – 2005 [8], Csiszár – 2005 [12], Hashimoto, Ibaraki, Imahori and Yagiura – 2006 [28], Oliveira, Vasconcelos and Alvarenga – 2006 [41], Dondo and Cerdá – 2006 [24], Fábry – 2006 [25], Repoussis, Tarantilis and Ioannou – 2006 [49], Ibaraki, Imahori, Nonobe, Sobue, Uno and Yagiura – 2008 [30] etc.

2 Evolutionary Algorithms

Evolutionary algorithms (EA) indicate evolutionary computation that is inspired by biological evolution. EA differ from more traditional optimization techniques in that they involve a search from a „population" of solutions called „individuals" not from a single one. Each individual represents a candidate solution for a given problem. Associated with each individual is also the fitness which represents the relevant value of objective function and it guides the search. EA works in iteration. The iteration of EA involves a competitive selection that carried out poor solutions. The solutions with high „fitness" are „recombined" with other solutions by swapping parts of a solution with another. Solutions can be also „mutated" by making a small change to a single element of the solution. Recombination and mutation are used to generate new solutions and in that way to find the best one. This process is continued until a fixed number of iterations has been reached or the evolution has converged. During the last 30 years evolved different main schools of EA: genetic algorithms, evolutionary programming, evolutionary strategy, differential evolution, ant colony optimization, immunology system method, scatter search, particle swarm, self organizing migrating algorithm etc.

EA work well at solving the unconstrained problem with continuous variables, but several specific methods have been proposed to solve constrained problems with discrete or integer variables. Both main objectives of constrained optimization using evolutionary techniques are to bring the individuals into feasible domain or exploring efficiently the feasible domain to find a solution as close as possible to optimum. The methods can be classified by Michalewicz and Schoenauer [39] into four categories:

Methods based on penalties functions which penalize unfeasible solutions

1. Methods which make a clear distinction between feasible and unfeasible solutions
2. Methods using special reproduction operators to preserve feasibility of solutions
3. Hybrid methods

Based on experiments e.g. Onwubolu and Babu [42], Sekaj [51] EA could be employed to deal with functions that are e.g. none-fractional type, defined at real, integer or discrete argument space, constrained, multiobjective, nonlinear, needle-in-haystack problems, NP problems etc. No additional information such a gradient, convexity etc. are needed. From what was mentioned earlier it is evident, that EA are applicable in general. EA are very free-defined. The only think is to set an objective function. In this article, the possibility of evaluation of objective function for VRPTW is presented.

3 The Formulation and Solution Principle of the Vehicle Routing Problem with Time Windows

As mentioned earlier, the vehicle routing problem with time windows is an extension of travelling salesman problem with additional constraints (demand of nodes, capacity limit of a vehicle, time windows). Further on, the following notation will be used:

Let $G = (V, A)$ be an undirected complete weighted graph, where:

- V represent the set of nodes $N \cup \{0\}$ where N represents the set of nodes (called supply nodes) that can be visited, $N = \{1, 2, ..., n\}$ and 0 represents the node of origin (called depot)
- A is the set of edge, where (i, j) are unordered pair of vertices for all $(i, j) \in V$, where matrix $\mathbf{D} = \{d_{ij} \geq 0 \mid i \in V, j \in V\}$, if $i \neq j$ then $d_{ij} > 0$ and if $i = j$ then $d_{ij} = 0$, represent the weights of the graph (direct connection between all pairs of nodes) and $d_{ij} > 0$ describes a shortest distance (in minutes) from the node i to node j through edge $(i, j) \in A$, $i \neq j$.

Let $[f_i, l_i]$ be the known time window for every i-th supply node from the set (for all $i \in N$), where the lower limit f_i indicates the earliest possible service time of the i-th supply node (if the earliest time is unbounded, then $f_i = 0$) and l_i indicates the latest possible service time of the i-th supply node (if the latest time is unbounded, then $l_i = M$)[2]. The service time of the i-th supply node including depot (for all $i \in V$) is t_i. Next, we consider that the demand of each supply node q_i (for all i N) is known. The need of the transfer is to move the required number of goods from depot to all supply nodes with the use of one vehicle (we don't consider fleet of vehicles) with the certain capacity v, where $q_i \leq v$, (for all $i \in N$).

[2] M represent a large prespecified positive number

The goal is to minimize the total travelled physical or time distance (route T) as a sequence of routes from depot to supply nodes, with respect to following criteria: the depot 0 is the initial node and also the final node for each route from which the demands of the supply nodes must be met within their time windows, the vehicle is allowed to wait for service when a vehicle arrives before the earliest limit, each supply node (except depot) is visited exactly once and the sum of all demands on route must not exceed the capacity of vehicle.

Further on, let us introduce two types of variables: the variable b_i that represent the real time of start of service of the i-th customer so that $b_i \geq f_i$ and the variable e_i that represent the real time of finishing the service of the i-th customer, so that $e_i \leq l_j$ (for all $i \in N$). The variable e_i is calculated as follows: real time of start of service of the i-th customer b_i plus time of service of the i-th node t_i, i. e. $e_i = b_i + t_i$ (for all $i \in N$).

We consider that the final route needs to satisfy the following criteria:

- The vehicles that arrived earlier than f_i are allowed to wait for service till the time f_i is reached ($b_i = f_i$), the waiting time is added to the total time corresponding route
- The supply nodes are included in the same route only in case that the savings $u_{ij} \geq 0$, the computation of savings is based on heuristic Clarke-Wright algorithm so that: $u_{ij} = d_{0j} + d_{i0} - d_{ij}$
- If the vehicle goes to the depot in the case of:
  - The capacity of a vehicle is exceeded $\Rightarrow$ the service time of the depot id added to the total time of the corresponding route (in practice, the next route will be realized on the some day)
  - Violating of the latest possible service time $\Rightarrow$ we don't take the service time of the depot into consideration (in practice, the next route will be realized on the next day)

Even through it is possible to formulate the above mentioned problem as integer (binary) programming problem (the problem involves a fleet of vehicles set that are operated from and to the depot, number of vehicles used is free or the capacity at least one vehicle is unlimited) and to solve it by exact methods (algorithms for solving integer programming problems), to date, there is no consistent optimization algorithm to solve it efficiently, so the use of alternative techniques seems to be more effective. Except of many classical heuristics the evolutionary techniques enable to find solution in an effective way.

VRPTW is a discrete optimisation problem. By solving, a natural representation of individual, represented by vector **j** (size n) that is known from genetic algorithm, can be used. Each supply node is originally assigned with a different integer value 1 to n. An individual **j** includes the permutation of those integer values that represents the order of visiting the supply nodes. Each individual is evaluated by fitness that represents the duration of the tour. To decode individual **j** into the route configuration and into route duration (fitness), we introduce a following structure in pseudocode:

Procedure Evaluate

Input: **D** matrix (size n+1 x n+1) of minimal distances (in minutes) between supply nodes and between depot and supply nodes (represented by the first row and column)

g vector of demand of each supply node (size n)

t vector of service time of each supply node including depot (size n+1)

f vector of first possible time to serve relevant supply node (size n)

l vector of last possible time to serve relevant supply node (size n)

v capacity of a vehicle

Output: *fitness* (duration of serving in minutes) including

b real time of the start of service the supply node in corresponding route

e real time of the finishing of service the supply node in corresponding route

z total time needed to serve corresponding supply nodes

k current load of vehicle (sum of the supply nodes that are served up to the moment)

U matrix (size n x n) of savings based on Clarke–Wright algorithm

$k = 0$

$z = 0$

$b = 0$

$e = 0$

Calculate **U**

Calculate b from depot till supply node **j**[1]

If $b \geq$ **f**[**j**[1]] then recalculate e, z, k directly from depot till supply node **j**[1]

Else recalculate b, e, z, k till supply point **j**[1] in case of waiting

Endif

for i=1 to n-1

Recalculate e, b, k directly till supply node **j**[i+1]

If $b \geq$ **f**[**j**[i+1]] $\wedge$ $e \leq$ **l**[**j**[i+1]] $\wedge$ k v $\wedge$ **U**[**j**[i],**j**[i+1]] $\neq 0$ then recalculate z from supply node **j**[i] directly till supply node **j**[i+1]

Elseif $b <$ **f**[**j**[i+1]] $\wedge$ $k \leq v$ $\wedge$ **U**[**j**[i],**j**[i+1]] $\neq 0$ then recalculate b, e, z, k till supply point **j**[i+1] in case of waiting

Elseif $e >$ **l**[**j**[i+1]] then $k = 0$, $b = 0$, recalculate b from depot till supply node **j**[i+1]

If $b \geq$ **f**[**j**[i+1]] then recalculate z, e, k till supply node **j**[i+1] through depot

Else recalculate b, e, z, k till supply node **j**[i+1] through depot in case of waiting

Endif

Elseif k ≥ v **U**[**j**[i],**j**[i+1]] = 0 then $k = 0$, recalculate b till supply node **j**[i+1] through depot

If b < **f**[**j**[i+1]] then recalculate b, e, z, k till supply node **j**[i+1] through depot in case of waiting

Elseif $e >$ **l**[**j**[i+1]] then $b = 0$, recalculate b from depot till supply node **j**[i+1]

If $b \geq$ **f**[**j**[i+1]] then recalculate z, e, k till supply node **j**[i+1] through depot

Else recalculate *b, e, z, k* till supply node **j**[i+1] through depot in case of waiting
Endif
Elseif recalculate *z*, *e*, *k* till supply node **j**[i+1] through depot
Endif
Endif
Endfor
Recalculate *d* from supply node **j**[n] to depot
Fitness = *z*
Endprocedure

The above procedure involves the use of many evolutionary techniques, e. g. genetic algorithms, evolutionary strategy, differential evolution, self organizing migrating algorithm etc. The authors' practical experiments that were presented in publications and at conferences pointed out usableness of presented approach [13], [15], [16] [17], [18]. The experiments show that the use of self organizing migrating algorithm and differential evolution give satisfactory results. The special approach that guaranties feasibility of solution was used by computation e.g. [13], [15], [17] with the setting of control parameters that is presented in e. g. [14], [15].

Conclusion

The vehicle routing problem with time windows belongs to NP-hard problems, so no algorithm has been known to solve the NP-hard problem in the polynomial time. The exact algorithms give exact solution but due to the computational complexity are applied only for relatively small problems. With the development of information technology the number of problems that can be solved by exact algorithms has increased. But this still does not reduce the computation time in order to be implemented in case of large problems, which usually occur in the real transportation. The alternative is, except for classical heuristics, the use of evolutionary algorithm, which can give after finite number of iteration "effective" solution. Although evolutionary algorithms are very free-defined, their use is conditioned by finding out in such a way an individual can be represented and decoded into route duration. Based on authors' experiments with solving the real data VRPTW by self organizing migrating algorithm and differential evolution, the use of presented approach enables to obtain good solution in an acceptable time.

Literature

[1] ANDROUTSOPOULOS, K. N. – ZOGRAFOS, K. G.: Solving the k-shortest path problem with time windows in a time varying network. In: *Oper. Res. Lett. 36(6)*, pp. 692-695 (2008).
[2] ANEJA, Y. P. – AGGARWAL, V. – NAIR, K. P. K.: Shortest chain subject to side constraints. In: *Networks 13/1983* 295-302.
[3] ASCHEUER, N. – FISCHETTI, M. – GRÖTSCHEL, M.: A Polyhedral Study of the Asymmetric Travelling

Salesman Problem with Time Windows. In: *Networks*, *36:2* (2000), pp. 69-79.
[4] BAKER, E. K.: An exact algorithm for the time-constrained traveling salesman problem. In: *Oper. Res. 31/1983*, pp. 938-945.
[5] BALL, M. O. et al., Eds.: *Handbooks in Operations Research and Management Science*. Vol. 8: Network Routing, Elsevier Science B.V., Oxford, 1995.
[6] BAR-YEHUDA, R. – EVEN G. – SHAHAR, S.: On Approximating A Geometric Prize-Collecting Traveling Salesman Problem With Time Windows. In: *ESA-2003, Lecture Notes in Computer Science* 2832: pp. 55-66, Springer 2003.
[7] BODIN, L., GOLDEN, B. – ASSAD, A., BALL, M.: The state of the art in the routing and scheduling of vehicles and crews. In: *Working Paper MS/S # 81-035*, September 1981.
[8] BRÄYSY, O. – GENDREAU, M.: Vehicle Routing Problem with Time Windows, Part I: Route Construction and Local Search Algorithms. In: *Transportation Science, Vol. 39/2005*, No. 1, pp. 104-118.
[9] CARLTON, W. B. – BARNES, J. W.: *Solving the Traveling Salesman Problem with Time Windows Using Tabu Search*. Institute of Industrial Engineering Transactions, 28, 1996.
[10] CARPANETO, O. – TOTH, P.: Some new branching and bounding criteria for the asymmetric traveling salesman problem. In: *Management Science, 26/1980*, pp. 736-743.
[11] CYRUS, J. P.: *The vehicle scheduling problem: Models, complexity and algorithms*. Ph.D. Thesis, Technical University of Nova Scotia, Department of Engineering, Halifax, NS, 1988.
[12] CSISZÁR, S.: Route Elimination Heuristic for Vehicle Routing Problem with Time Windows. In: *Acta Polytechnica Hungarica, Vol. 2*, No. 2, 2005.
[13] ČIČKOVÁ, Z.: Riešenie úlohy obchodného cestujúceho evolučnými technikami. (Solving the problem of travelling salesman by means of evolutionary techniques). In: *INPROFORUM 2007*: inovace–podniky–regiony–organizace: sborník příspěvků z mezinárodní vědecké konference: 27. - 28. listopadu 2007 České Budějovice. České Budějovice: Ekonomická fakulta Jihočeské univerzity v Českých Budějovicích, 2007. ISBN 978-80-7394-016-4, s. 96-101.
[14] ČIČKOVÁ, Z.: Nastavenie riadiacich parametrov algoritmu diferenciálnej evolúcie. (Setting regulating parameters of differential evolution algorithm). In: *International Seminar of Young Researchers of the Department of Econometrics Faculty of Informatics and Statistics, Higher School of Economics in Prague and the Department of Operations Research and Econometrics, Faculty of Econometric Informatics, EUBA in Bratislava*: Collection of papers: Prague, 2. - 4. December 2008, Higher School of Economics in Prague. Praha: Nakladatelství Oeconomica, 2008. ISBN 978-80-245-1405-5.
[15] ČIČKOVÁ, Z. – BREZINA, I.: SOMA application to the traveling salesman problem. In: *Mathematical methods in economics 2006: proceedings of the 24th international conference*: 13th - 15th September 2006, Plzen: Czech Republic. ISBN 80-7403-480-5, pp. 117-121.
[16] ČIČKOVÁ, Z. – BREZINA, I.: An evolutionary approach for solving vehicle routing problem. In: *Quantitative methods in economics: multiple criteria decision making XIV: proceedings of the international conference*: 5th - 7th June 2008, High Tatras, Slovakia; Bratislava: IURA EDITION, 2008. ISBN 978-80-8078-217-7, pp. 40-44.
[17] ČIČKOVÁ, Z. – BREZINA, I. – PEKÁR, J.: SOMA for solving the vehicle routing problem with time windows In: *Zbornik radova: SYM-OP-IS 2008*, XXXV Simpozijum o operacionim istraživanjima, 14 - 17 septembar, 2008, Soko Banja. - Beograd: Univerzitet u Beogradu, 2008. ISBN 978-86-7395-248-2, pp. 305-308.
[18] ČIČKOVÁ, Z. – BREZINA, I. – PEKÁR, J.: Riešenie úlohy rozvozu materiálu s časovými oknami samo-organizujúcim sa migračným algoritmom. (Solving the Vehicle Routing Problem with Time Windows by Self-organising Migration Algorithm). In: *AIESA – Building of Knowledge-Based Society*: The Twelfth International Scientific Conference [of Doctoral Students]: Bratislava, 19. - 21. November 2008: 40th Anniversary of the Foundation of the Faculty of Economic Informatics, EUBA in Bratislava. Bratislava: Iura Edition, 2008. ISBN 978-80-8078-233-7.
[19] DESAULNIERS, G. – VILLENEUVE, D.: The shortest path problem with time windows and linear waiting costs. In: *Transportation Science 34(3*)/2000, pp. 312-319.
[20] DESROCHERS, M., DESROSIERS, J., SOLOMON, M.: A new optimization algorithm for the vehicle routing problem with time windows. In: *Oper. Res. 40/1992*, pp. 342-353.
[21] DESROSIERS, J. – DUMAS, Y. – SOLOMON, M. M. and SOUMIS, F.: Time constrained routing and scheduling. In: *M. O. Ball, T. L. Magnanti, C. L. Monma, and G. L. Nemhauser, editors, Network Routing, Handbooks in Operations Research and Management Science 8,* North-Holland, Amsterdam, 1995, pp. 35-139.

[22] DESROSIERS, J. – SOUMIS, F. – DESROCHERS, M.: Routing with time windows by column generation. In: *Networks 14/1984*, 545-565.

[23] DONATI, A. V. – MONTEMANNI, R. – GAMBARDELLA, L. M. – RIZZOLI, A. E.: Integration of a robust shortest path algorithm with a time dependent vehicle routing model and applications. In: *Proceedings of CIMSA 2003- International Symposium on Computational Intelligence for Measurement Systems and Applications*. 2003.

[24] DONDO, R. – CERDÁ, J.: An MILP framework for dynamic vehicle routing problems with time windows. In: *Lat. Am. Appl. Res., Vol. 36/2006*, No. 4, pp. 255-261.

[25] FÁBRY, J.: *Dynamické okružní a rozvozní úlohy.* (Dynamic Circular and Distribution Jobs). Praha: VŠE-FIS, 2006.

[26] GENDREAU, M. – GUERTIN, F. – POTVIN, J. – TAILLARD, E.: Parallel tabu search for real-time vehicle routing and dispatching. In: *Transportation Science*, Vol. 33/1999, No. 4, pp. 381-390.

[27] HANSEN, P.: Bi-criterion path problems. In: *Multiple Criteria Decision Making: Theory and Applications,* LNEMS 177 (G. Fandel and T. Gal, Eds.), 1980. Berlin: Springer-Verlag, pp. 109-127.

[28] HASHIMOTO, H. – IBARAKI, T. – IMAHORI, S. – YAGIURA, M.: The vehicle routing problem with flexible time windows and traveling times. In: *Discrete Applied Mathematics*, Volume 154/2006, Issue 16), pp. 2271-2290.

[29] HOMBERGER, J. – GEHRING, H.: A two-phase hybrid metaheuristic for the vehicle routing problem with time windows. In: *European Journal of Operational Research 162* (1)/2005, pp. 220-238.

[30] IBARAKI, T. – IMAHORI, S. – NONOBE, K. – SOBUE, K. – UNO, T. – YAGIURA, M.: An Iterated Local Search Algorithm for the Vehicle Routing Problem with Convex Time Penalty Functions. In: *Discrete Applied Mathematics,* 156/2008, pp. 2050-2069. etc.

[31] JAFFE, J. M.: Algorithms for Finding Paths with Multiple. Constraints. In: *Networks*, Vol. 14/1984, pp. 95-116.

[32] JANÁČEK, J.: *Optimalizace na dopravních sítích.* (Optimisation in Transportation Networks). Žilina: EDIS, 2003, s. 248, ISBN 80-8070-031-1.

[33] JOKSCH, H. C.: The shortest route problem with constraints. In: *J. Math. Anal. Appl.*, 14/1966, pp. 191-197

[34] KOHL, N. – DESROSIERS, J. – MADSEN, O. B. G. – SOLOMON, M. M. – SOUMIS, F.: 2-Path Cuts for the Vehicle Routing Problem with Time Windows, In: *Transportation Science*, 33(1), pp. 101-116, (1999).

[35] LANGEVIN, A. – DESROCHERS, M. – DESROSIERS, J. – GELINAS, S. – SOUMIS, F.: A two-commodity flow formulation for the traveling salesman and the makespan problems with time windows. In: *Networks,* 23/1993, pp. 631-640.

[36] LAPORTE, G. – NOBERT, Y.: Exact algorithms for the vehicle routing problem. In: *Ann. Discrete Math*. 31/1987, pp. 147-184.

[37] LAU, H. C. – SIM, M. – TEO, K. M.: Vehicle routing problem with time windows and a limited number of vehicles. In: *European Journal of Operational Research*, Vol. 148/2003, No. 3, pp. 559-569.

[38] MAGNANTI, T.: Combinatorial optimization and vehicle fleet planning: Perspectives and prospects. In: *Networks*, Vol. 11, 1981.

[39] MICHALEWICZ, Z. – SCHOENAUER, M.: Evolutionary Algorithms for constrained parameter optimization problems. In: *Evolutionary Computation*, Vol. 4, No. 1, 1996.

[40] MINOUX, M.: Plus courts chemins avec contraintes: algorithmes et applications. In: *Annales des 7~l~communications* 30/1975, pp. 383-394.

[41] OLIVEIRA, H. C. B. – VASCONCELOS, G. C. and ALVARENGA, G. B.: Reducing Traveled Distance in the Vehicle Routing Problem with Time Window using a Multi-Start Simulated Annealing. In: *International Joint Conference on Neural Networks* (IJCNN'06), Vancouver, Canada, 2006.

[42] ONWUBOLU, G. C. – BABU, B. V.: New Optimization Techniques in Engineering. In: *Studies in Fuziness and Soft computing*, Berlin-Heidelberg : Springer-Verlag, Volume 14, 2004.

[43] OR, I.: *Traveling Salesman-Type Combinatorial Problems and Their Relation to the Logistics of Regional Blood Banking*. Ph.D. Thesis, Northwestern University, Evanston, Illinois, 1976.

[44] PEŠKO, Š: *Optimalizácia rozvozov s časovými oknami*. (Optimisation of Distributions with Time Windows). Conference "Cybernetics – History, Prospects, Theory and Practice", Conference Proceedings, Žilina, pp. 76-81, ISBN 80-967609-7-1, 2002.

[45] PEŠKO, Š: *Differential Evolution for Small TSPs with Constraints.* Proceedings of the fourth Internattional Scientific Conference: Challenges in Transport and Communications, Part III, September 14 - 15, Pardubice, Czech Republic, ISBN 80-7194-880-2, pp. 989-994, 2006.

[46] POTVIN, J.-Y. – ROUSSEAU, J.-M.: *A parallel route building algorithm for the vehicle routing and*

scheduling problem with time windows. In: Eur. J. Oper. Res. 66/1993, pp. 331-340.

[47] PSARAFTIS, H. – SOLOMON, M. – MAGNATI, T., and KIM, T.: *Routing and scheduling on a shoreline with release times*. In: Management Science, Vol. 36/1990, pp. 212-223.

[48] QURESHI, A. G. – TANIGUCHI, E. – YAMADA, T.: *Elementary shortest path problem with resource constraints and time dependent late arrival penalties*. In: Japan Society of Civil Engineers, Vol. 63/2007, No. 4, pp. 579-590.

[49] REPOUSSIS, P. P. – TARANTILIS, Ch. D. – IOANNOU, G.: *A Hybrid Metaheuristic for a Real Life Vehicle Routing Problem*. In: Numerical Methods and Applications 2006, pp. 247-254.

[50] SAVELSBERGH, M. W. P.*: Local search in routing problems with time Windows*. In: Annals of Operations Research 4(1985/6), pp. 285-305.

[51] SEKAJ, I.: *Evolučné algoritmy a ich použitie v praxi.* IRIS Bratislava, s. 157, ISBN 80-89018-87-4, 2005.

[52] TAN, K. C. – LEE, L. H. – ZHU, Q. L. – OU, K.: *Heuristic methods for vehicle routing problem with time Windows.* In: Artificial Intelligence in Engineering, Vol. 15/2001, No. 3, pp. 281-295.

[53] TSITSIKLIS, J. N.: *Special Cases of Traveling Salesman and Repairman Problems with Time Windows*. In: Networks, Vol. 22/1992, No. 3, pp. 263-282.

[54] VACIC, V. – SOBH, T. M.: *Vehicle routing problem with time windows*. In: International Scientific Journal of "Computing", Research Institute of Intelligent Computer Systems, Ternopil National Economic University, Vol. 3/2004, Issue 2.

[55] ZHANG, Y. – TANG, L.: *Solving Prize-Collecting Traveling Salesman Problem with Time Windows by Chaotic Neural Network*. In: Lecture Notes In Computer Science, Vol. 4492/2007, pp. 6-71.