ON THE NATURE/REASONS FOR THE LONG-TERM NON-LINEARITY OF ECONOMIC DEVELOPMENT

LADISLAV ANDRÁŠIK[1]

O povahe/dôvodoch dlhodobej nelinearity ekonomického rozvoja

Abstracts: *There are several different approaches to understanding the peculiar behaviour of economies in the global economic literature. The best known of these is the business cycle theory, which we have referred to as the "conventional theory of the non-linear behaviour of economies" to better distinguish it from the others. For a typical graphical representation, the well-known sine smooth graph is usually used. There are also newer approaches using mathematical, econometric and ecological methods. However, the approach of the author of this essay differs from all others in ontological and methodological aspects. In this paper, among other things, we present an extension of Goodwin's growth cycle model, in which the capacity utilization rate is treated as a new variable in a modified Lotka-Volterra system of differential equations (1956) to the study of distribution cycles. This approach is intended to clarify the links between demand, the labour market and capital accumulation in a model in which a cyclical pattern is established between the employment rate, the profit share and the capacity utilisation rate. The resulting model is then examined qualitatively and the conditions for a limit cycle are explored. Finally, the model is tested on the US economy with quarterly data from 1967 to 2016. The model has a system of differential equations for the utilization rate, the profit share and the employment rate. The second example comes from a slight modification of the original duopoly model of Cournot. Next, in the third example, we visualize Mordechai Ezekiel's famous spider's web theorem using Excel.*

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JEL Classification: C13, E11, C61, E32, F02, F43, F47, O11, O41.

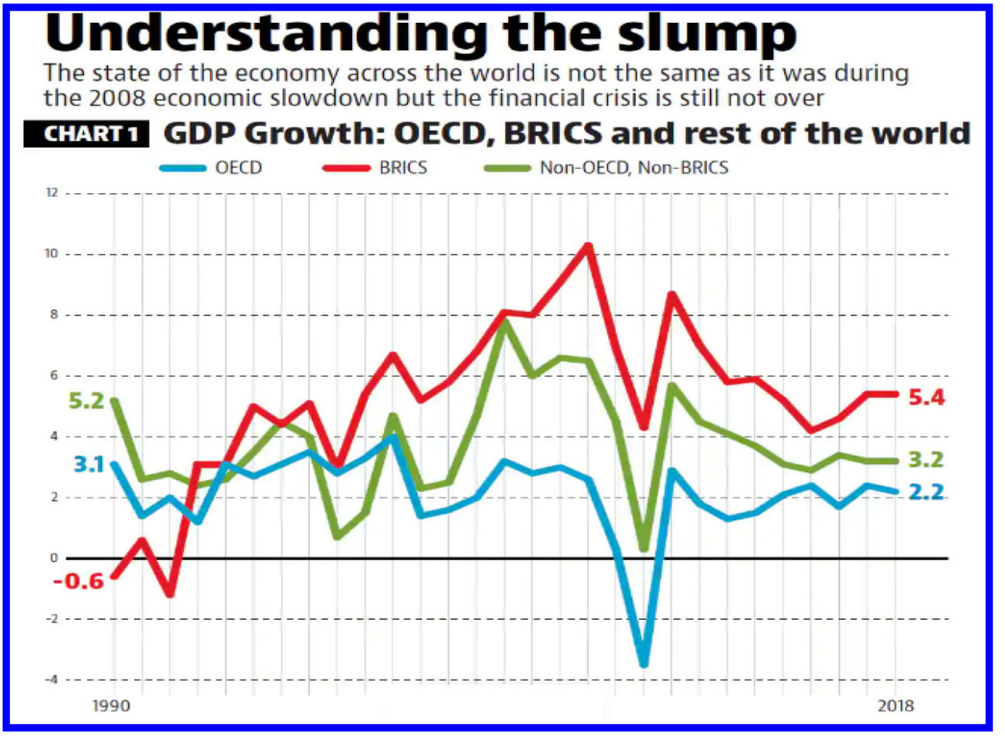
1 Introduction

In scientific thinking about the entity of the economy in objective reality as a whole, future scientists are inclined to see this entity as a *complexly evolving social organism* and not to imagine it as a simple mechanical system. In this case, the causality and presence of non-linearity or shocks in development is sometimes realised through the alternation between *structural stability* and *structural instability*. If structural stability is disturbed by some perturbation at birth, structural instability may arise perhaps by a different topological quality (shape of trajectories), perhaps different from the conventional smooth pattern of cycles. The main reason may lie in the fact that the economy is made up of several qualitatively different groups of actors as "economic species". It is a matter of cause that very complex types of interactions arise between all such populations that are not described otherwise than in ecological approaches and terminology, such as symbiosis[2], competition, cooperation, predation (very interesting predator-prey interactions), and others. Sometimes a situation arises where large and very powerful players among all the participating populations in an economy behave in a very unpleasant and/or harmful way for the others. A vivid example of such behaviour is the global slump caused by the bankruptcy of Lehman Brothers. On 15 September 2008, the bankruptcy of Lehman Brothers caused the Dow Jones Industrial Average to fall by 4.5% overnight, which at the time was the biggest fall since the attacks of 11 September 2001. It signalled the limitations of the government's ability to manage the crisis and caused widespread financial panic. Money market mutual funds, a key source of credit, saw runs to prevent losses and the interbank lending market tightened, putting banks at risk of imminent collapse. The government and the Federal Reserve responded with several extraordinary measures to contain the panic. To illustrate the impact of this event on the world economy, we do

[2] The term "symbiosis" encompasses a wide range of interactions between species, but usually refers to three main types: *mutualism, commensalism and parasitism. Mutualism* is a symbiotic interaction in which both or all individuals benefit from the relationship. Mutualism can be considered obligate or facultative.

not present a smooth cycle of the sine wave, but the sawtooth plots shown in Figure 1.

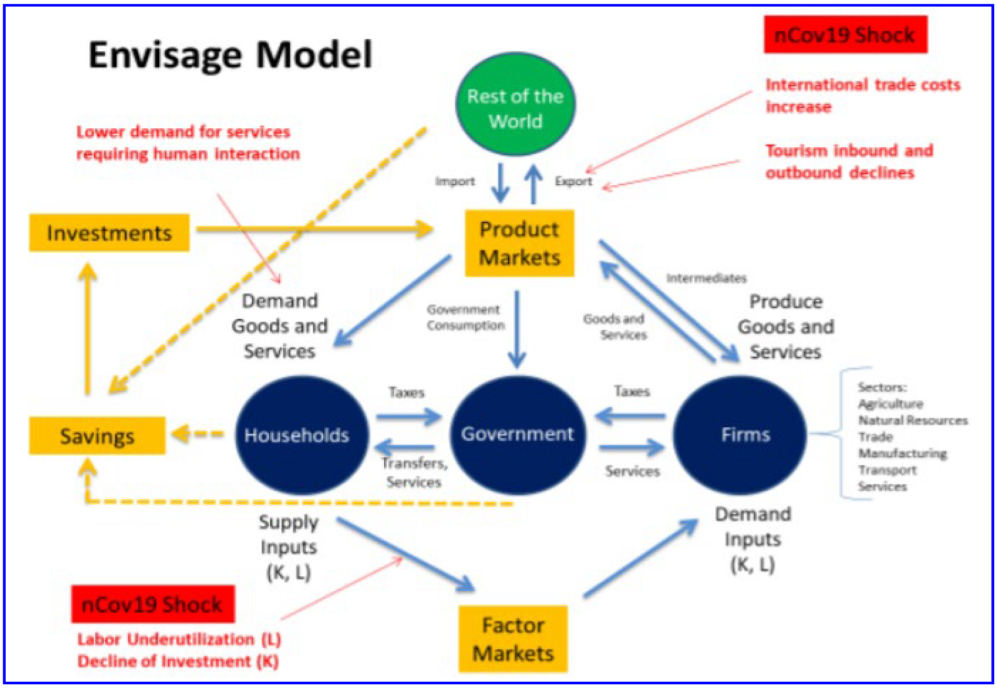
Figure 1: Sawtooth GDP growth after the Lehman Brothers bankruptcy



Source: https://www.hindustantimes.com/analysis/11-years-after-the-lehman-collapse-crisis-continues/story-RGUvD3cfiQModJlaU8nqkO.html

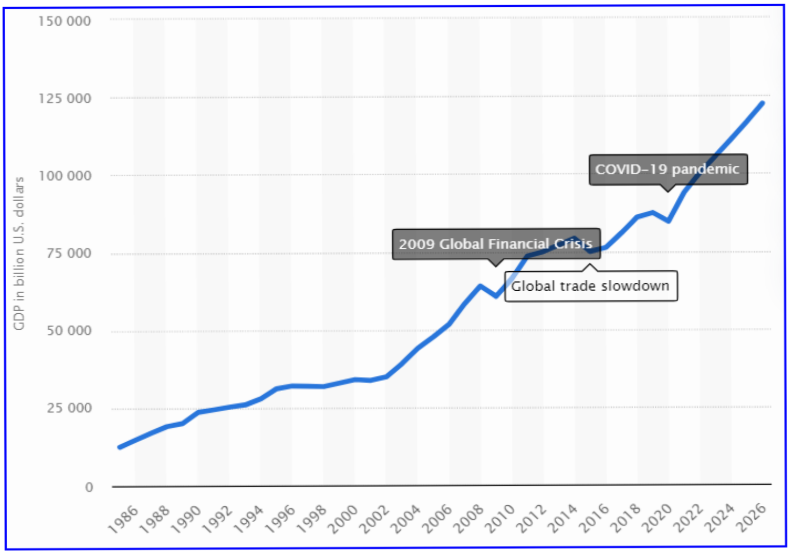
However, various shorter in time interval exogenous occurrences may arise in some local area, for example: a super-volcano eruption, a large earthquake, a wildfire, an extraordinary flood, a Spanish flu pandemic, a Covid 19 pandemic as at present, etc. In all such cases, their adverse effects disappear after some time. To illustrate such cases, we give for example the impact of Covid 19 pandemic on the development of the world economy, Fig. 2, and the GDP of the world economy, Fig. 3.

Figure 2: Implications of COVID-19 implemented in the Envisage model.



Source: https://openknowledge.worldbank.org/bitstream/handle/10986/33605/The-Potential-Impact-of-COVID-19-on-GDP-and-Trade-A-Preliminary-Assessment.pdf

Figure 3: Global GDP at current prices from 1985 to 2026 with two slowdowns (USD billion)

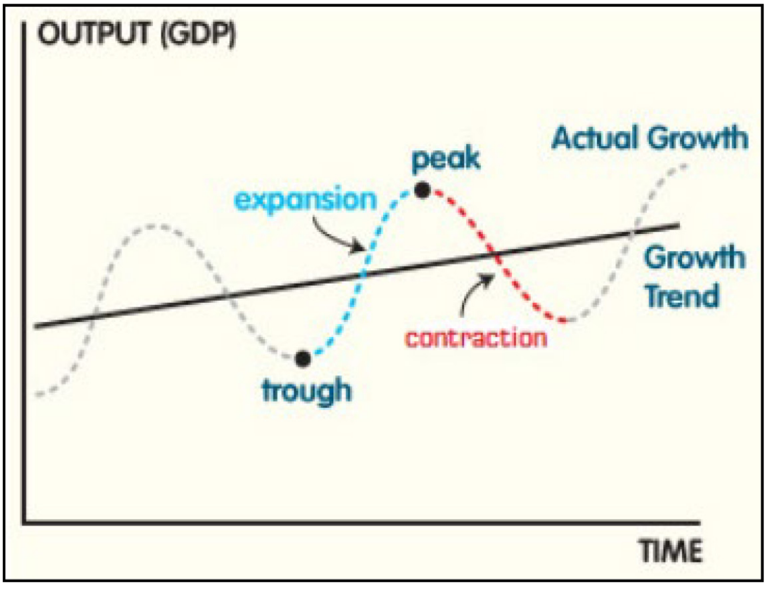


Source: https://www.statista.com/statistics/268750/global-gross-domestic-product-gdp/

2 Formal (mathematical and computational) approach to long-term economic development

In mathematics, such interactions give rise to very different topological forms of evolving continuous and/or discrete trajectories: among them, the smoothest is the smooth curve of the conventional business cycle visualized as a goniometric sine or cosine function; others are homoclinic tangles, invariant attracting and/or repelling closed curves, deterministic chaos, and several others. Unfortunately, if one is concerned with the behavior of economies over long periods of time, one can only find the simplest form of cycles in the mainstream economic literature in the form of the smoothly evolving cosine plot of Fig. 4. In objective reality, however, the economy evolves in a more complex way, among other things, often in the form of a *sawtooth-like* graph, Fig. 5.

Figure 4: Business cycles: the phases of the business cycle evolve over time in relation to GDP as waves, with an expansion leading to a peak followed by a contraction.



Source: https://courses.lumenlearning.com/baycollege-introbusiness/chapter/reading-the-business-cycle-definition-and-phases/

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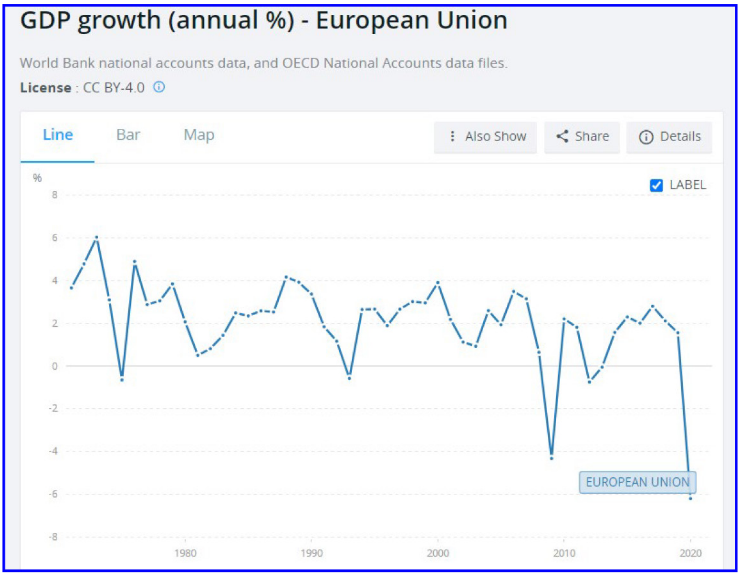
The principle of long-term fluctuation of the economy in objective reality based on ecological ideas remains valid even if this economy is generated only by populations of two economic species. This principle in virtual reality can be very successfully constructed on the basis of Goodwin's theory of cyclical growth. We use STELLA software to create a virtual economy based on Goodwin's formulas. In the author's opinion, Richard Goodwin was the first economist to discover these similarities. In fact, there is at least one level of interaction between a group of capitalists and a group of workers, with the capitalists acting as predators and the workers as prey (and/or vice versa?). This narrative comes into play after a contract is made between the authentic capitalist and the worker.

Figure 5: GDP growth path (% p.a.) for the EU as a whole



Source: World Bank Statistics https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=EU

The capitalist tries to extract as much surplus-value as possible from the worker, that is, he tries to catch up with (prey on) a large amount of worker labor. The worker, on the other hand, wants to shirk from the capitalist and tries to minimize his surplus value as much as possible. This descriptive process is thus the classical process of pursuit and avoidance known in differential game theory. Such processes are very difficult to compare from an analytical mathematical point of view. This is also correct for the analytically solved problem in Richard Goodwin's essay (1967). On the other hand, the possibilities offered by the current simulation software in PS allow us to solve this tangled mathematical problem without the need for complex mathematical erudition in solving differential equations. Moreover, it is obvious that scientists, researchers and/or students can in this way carry out experiments in a virtual model (virtual laboratory) of the class struggle between capitalists and workers. The author's aim is to show such possible experiments that will help the reader in his/her search for answers to the questions "What happens if...?"

In the next section of this essay, the author shows a simple case of Goodwin's model of class struggle in a conventional mathematical way. He then constructs a relatively simple model of the predator-prey model that looks like the Lotka-Volterra model. In the next section, the author constructs the capitalist-worker model in STELLA as mentioned earlier. Then, some results are exposed of several experiments with variable inputs to the model, and the author notes comments on the "dialogues" with STELLA. He first presents Goodwin's analytical model of class struggle.

To illustrate that the mathematics for this analysis is a very sophisticated affair indeed, we give an analytical description of the workers' struggle for higher wages. The demonstrated cause is Richard Goodwin's (1967) model. In his essay, he depicted a non-linear model of the economic cycle inspired by the Lotka-Volterra equations used in biological predator-prey scenarios, based on Marx's traditional model of class struggle. Goodwin's model appears to have been developed to demonstrate the cyclical nature of the capitalist market economy, using the contradictory relationship between employment and the wage share of the living economy. However, Goodwin's model is not entirely identical to Marx's mental model of class struggle, because in Goodwin's case the relationships are not actually as contradictory as they may sound at first hearing: 'class struggle' and 'predator-prey' can conjure up harsh images of antagonistic reaction. At first glance, the case makes the impression that Goodwin's approach is no more drastic than standard economic thinking in the second half of the 20th century in the thought segment of system dynamics theorizing.

EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 – 73
https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

3 Creating models in computing devices and their simulation runs in STELLA and Excel

The basic verbal features of Goodwin's model look like this:

1 high employment causes wage inflation, which can,

2 increase the share of workers' wages in output, which in turn will,

3 reduce the profits of capitalists, and thus reduce future investment and output, which in turn will,

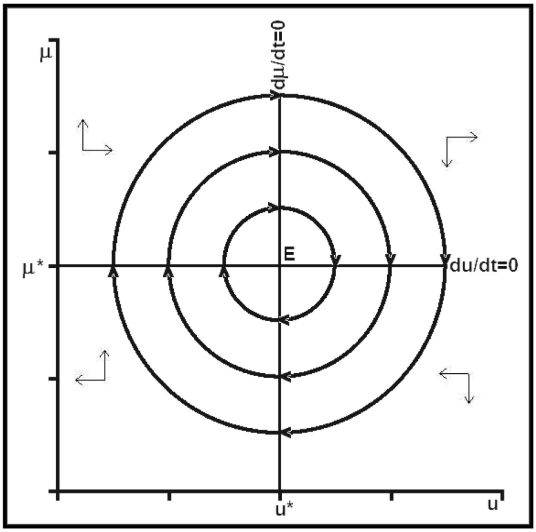
4 reduce labour demand, i.e. employment, and consequently lead to

5 lower wage inflation or even deflation and thus reduce the workers' wage share, but when the workers' wage share falls, then

6 profits increase and, in parallel with them, it leads to investment growth, higher investment will

7 lead to higher employment, thereby improving the negotiating power of workers and consequently leading to wage growth. It is immediately clear that such a process must be cyclical. As is well known, Goodwin adds other exogenous components of growth - namely, growth in labour supply and growth in productivity - to better illustrate this process. In Goodwin's model, there are two social classes of income earners, similar to Marx's approach: - a first class of wage-earning workers and - a second class of profit-earning capitalists. For the sake of space in this essay, we omit Goodwin's formulas and rely only on the formulas used in our STELLA model, which are similar to the Lotka-Volterra equations. These equations have vortex dynamics of the type shown in the graph in the following Fig. 6.

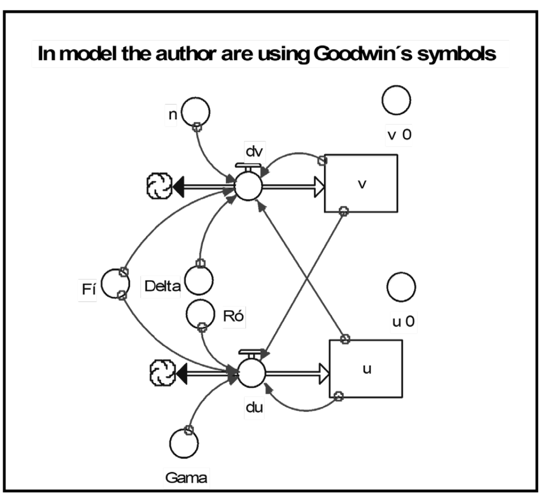
Figure 6: Goodwin variables: Goodwin's variables: Vortex dynamics in wage share and employment



Source: author

In the hand sketch in Fig. 6, *each* trajectory in the state space (u, μ) is a closed orbit around an equilibrium state denoted as E = (u*, μ*). Thus, as can be seen, we have cyclic dynamics for u and μ. The clockwise direction of the closed trajectories in Fig. 6 and the equilibrium values of (u*, μ*) and the isoclines are easily found from Goodwin's differential equations.

Figure 7: Basic blocks of the virtual economy model in STELLA



Source: author's calculations

EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 — 73
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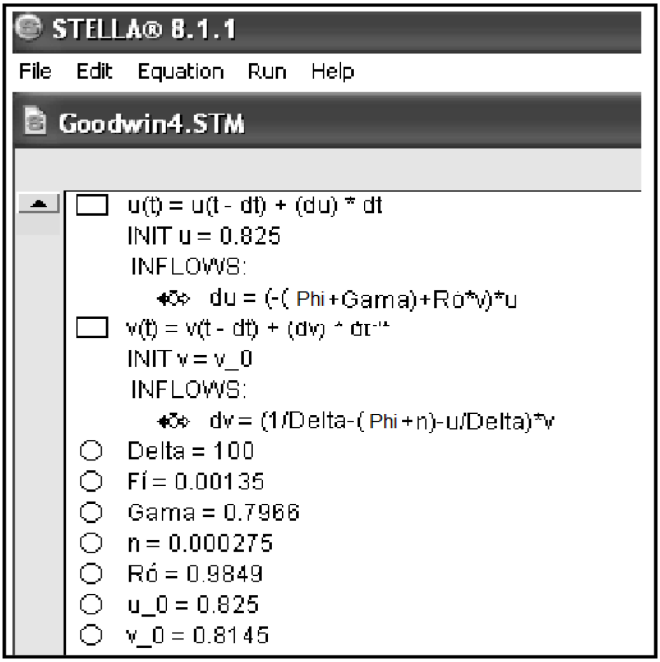
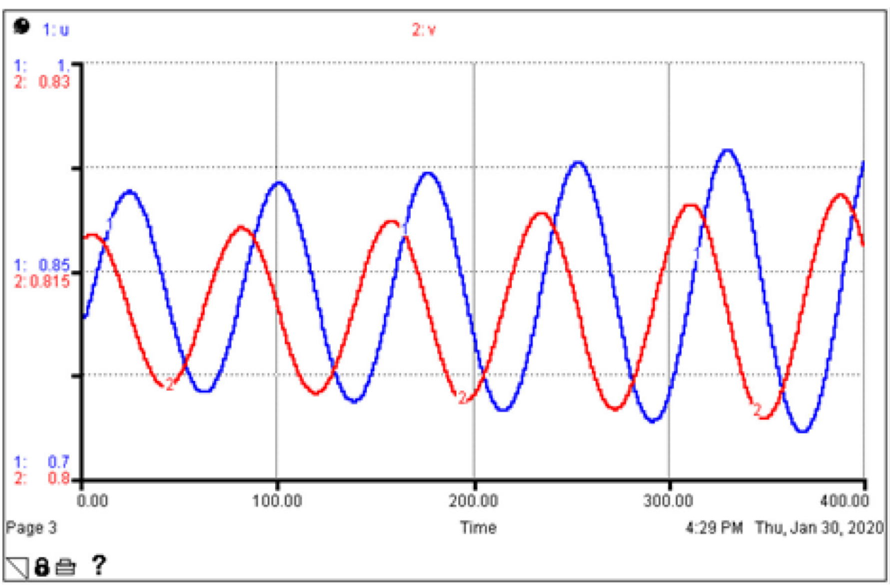
Figure 8: Main field of the model (field fills in Figure 7)



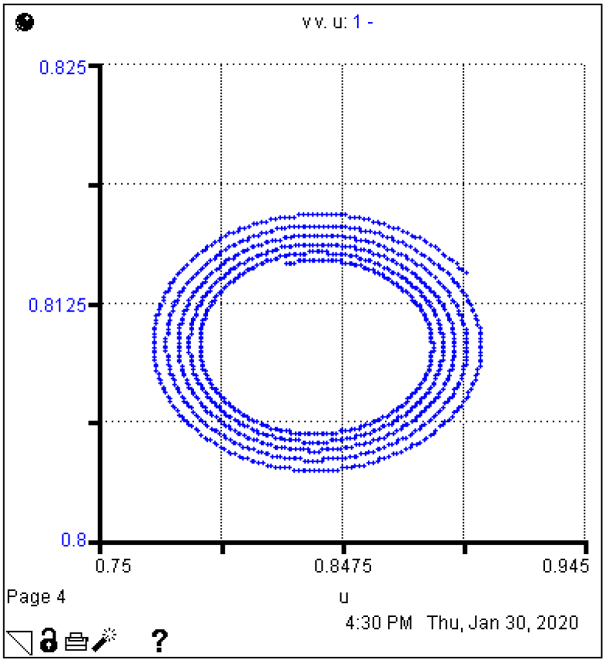
Source: author's calculation

Figure 9: Scatter plot of the simulation of the variables v and u in the STELLA environment



Source: author's calculations

Figure 10: Snapshot of the cyclic evolution of the variables v and u in time steps in STELLA



Source: author's calculations

Surprisingly, even Auguste Cournot's famous duopoly model can be interpreted in the form of an ecological mathematical model. There are two competitors, producers for a constant quantity of money supply, and both want to maximize their profits when the price is p = quantity of money/quantity of aggregate outputs. And similarly Mordecai Ezekiel's Cobweb model between supply curves along with demand and price in the virtual market. We first present the Cournot duopoly result obtained in STELLA and then the Cobweb result in Excel. Cournot's duopoly model is another very suitable case to demonstrate economic theory construction with the help of CI in the field of long-run fluctuations in virtual economies. We can say that it is perfectly suited for the initial stage of building economic theory with the help of CI. By the way, in my opinion, Cournot was the first mathematician and economist to apply mathematics in the field of economics, not necessarily to achieve numerical accuracy in a predictive way, but rather to provide a clearer formulation of economic relationships. This is the approach we have called *qualitative computational economics because,* in parallel with mathematics, computational intelligence must now dominate economic theory.

EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 — 73
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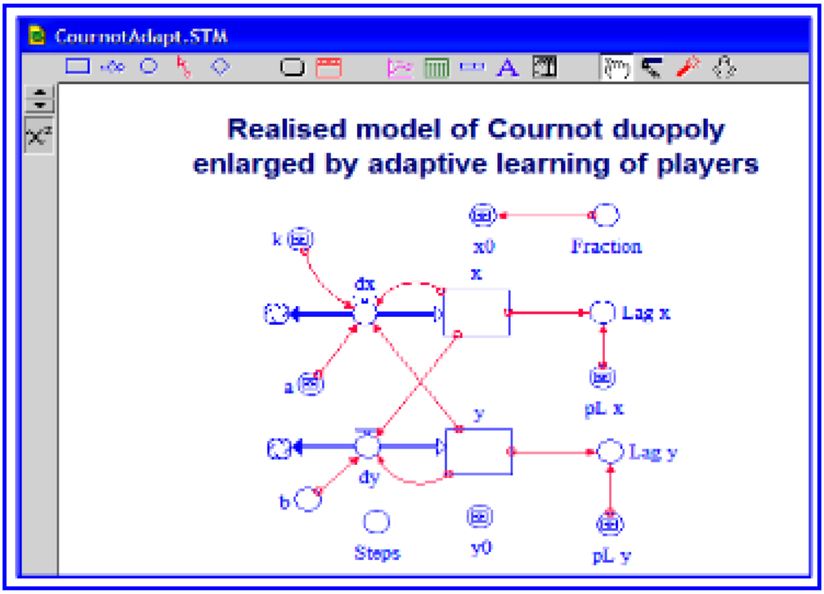
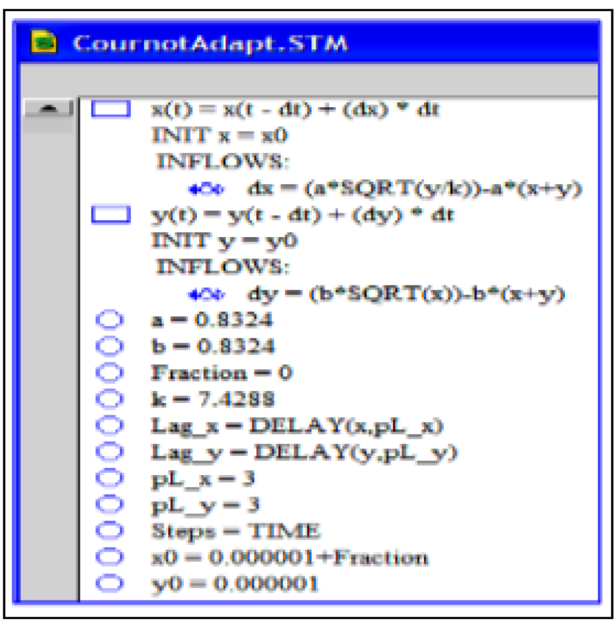
Figure 11: Basic schematic of the extended Cournot duopole model at the map/model level of STELLA 8.1



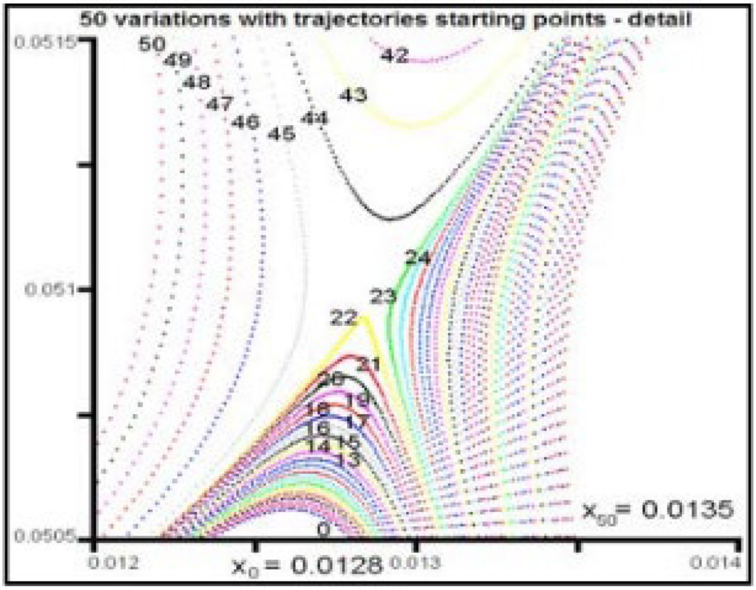
Source: author's calculations

Figure 12: Fulfillment of the basic Cournot duopoly scheme (main scheme) from Fig. 11



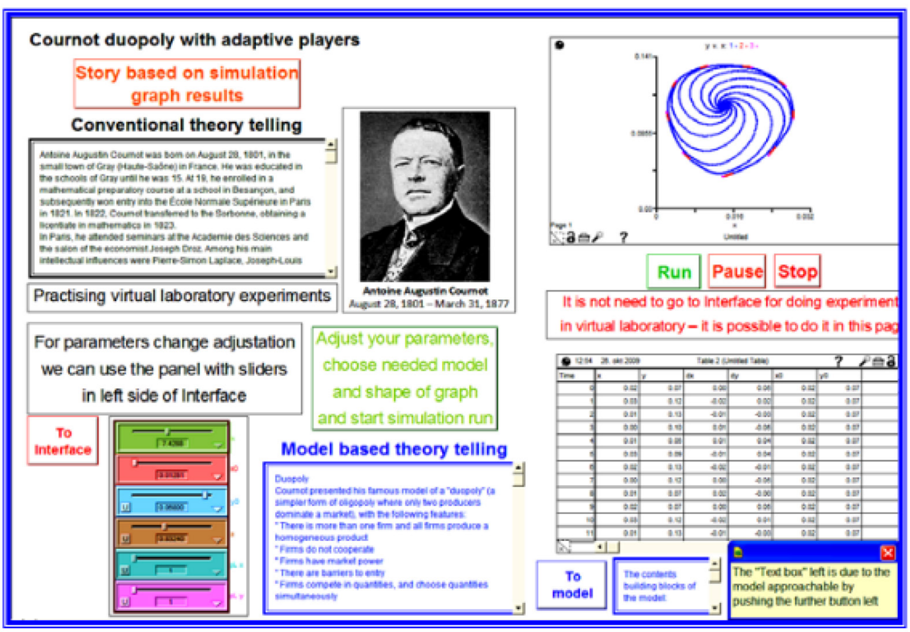
Source: author

Figure 13: Sensitivity of the model to the initial value of the STELLA trajectory simulation



Source: author's calculations

Figure 14: Implementation of storytelling based on the well-known duopoly theory (story tracking progress is halfway through)



Source: author

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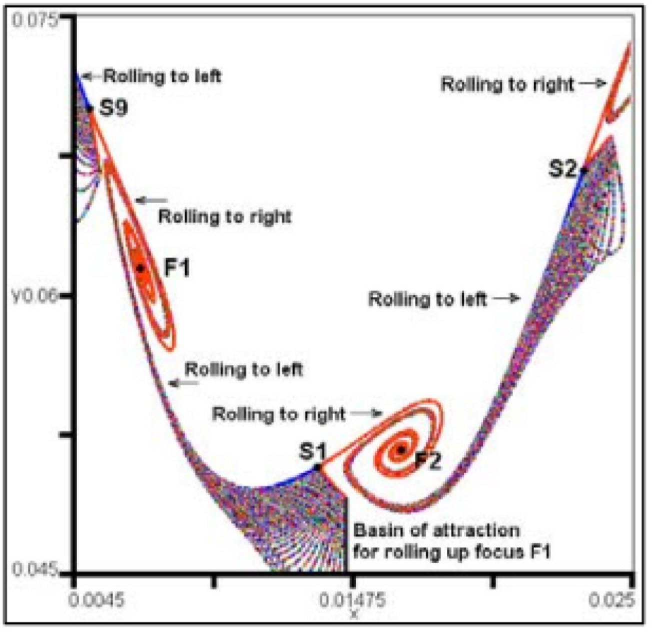
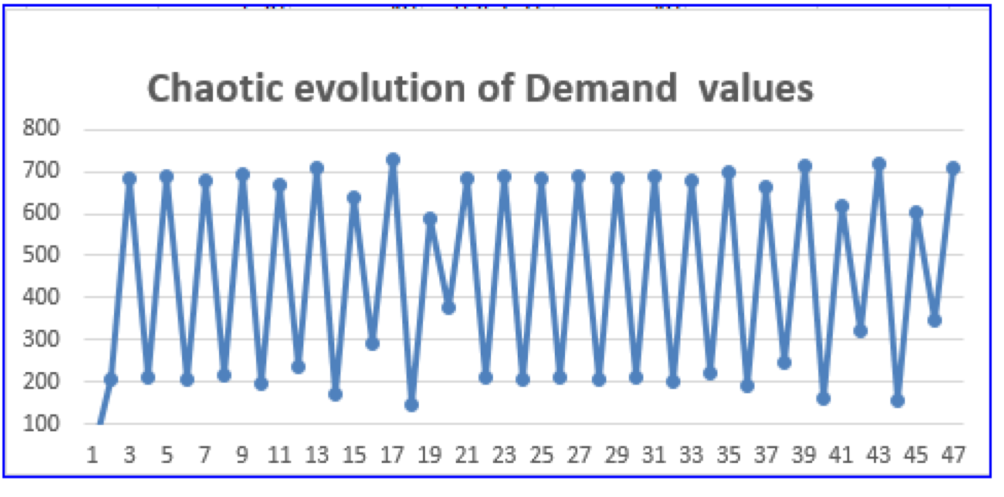
Figure 15: Visualisation of the change in rolling direction affected by forking in STELLA



Source: author's calculations

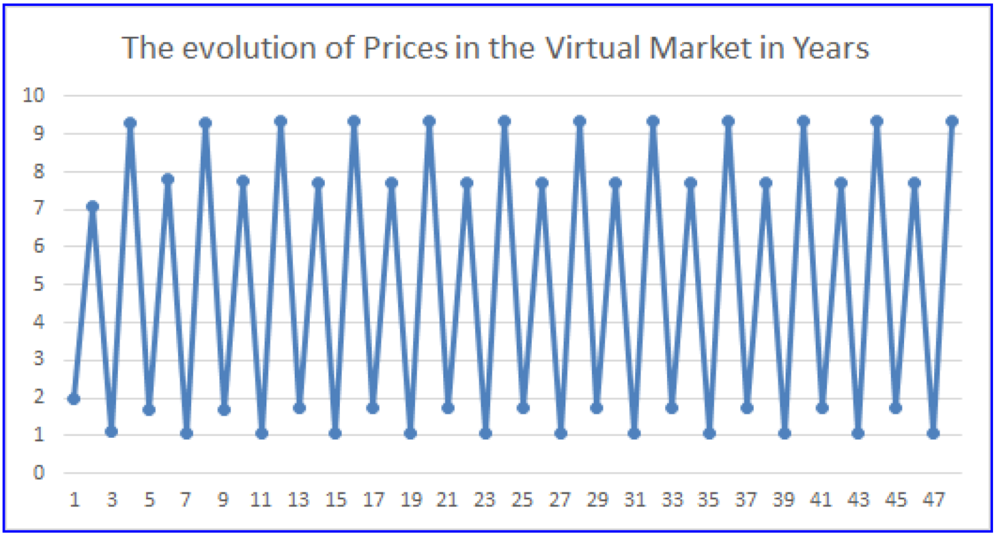
After experimenting with our duopoly models in STELLA, we create a spider web model of demand, supply and price in Excel and run a simulation with it.

Figure 16: Sawtooth price evolution in a virtual market where labour supply is a backward curve



Source: author's calculations

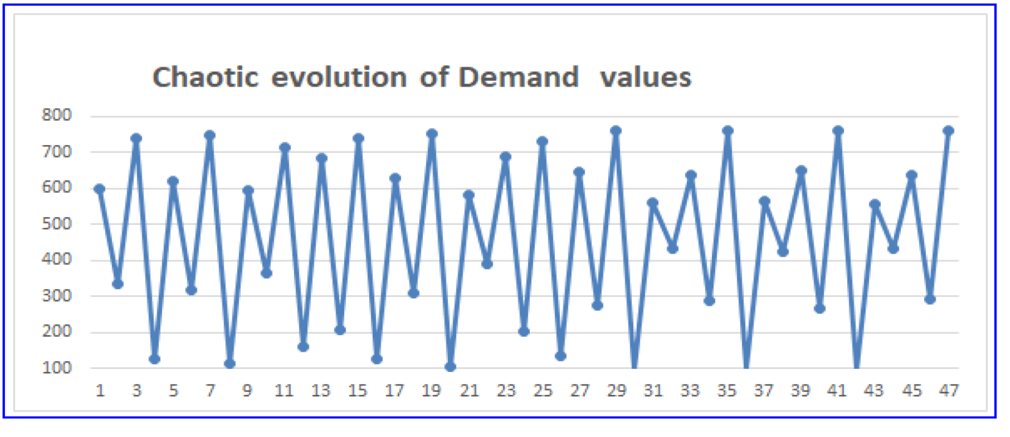
Figure 17: Pillar evolution of demand in a virtual market where labour supply is a backward curve



Source: author's calculations

The snapshot in Figure 19 shows the case where deterministic chaos is generated around four points (two on the demand line and two on the backwardation curve of the supply band). In the next slide of Fig. 18, the situation has changed with a slightly reformed supply curve and the chaos becomes more complex.

Figure 18: Sawtooth-shaped evolution of demand with more chaos: a slightly altered backward bending curve



Source: author's calculations

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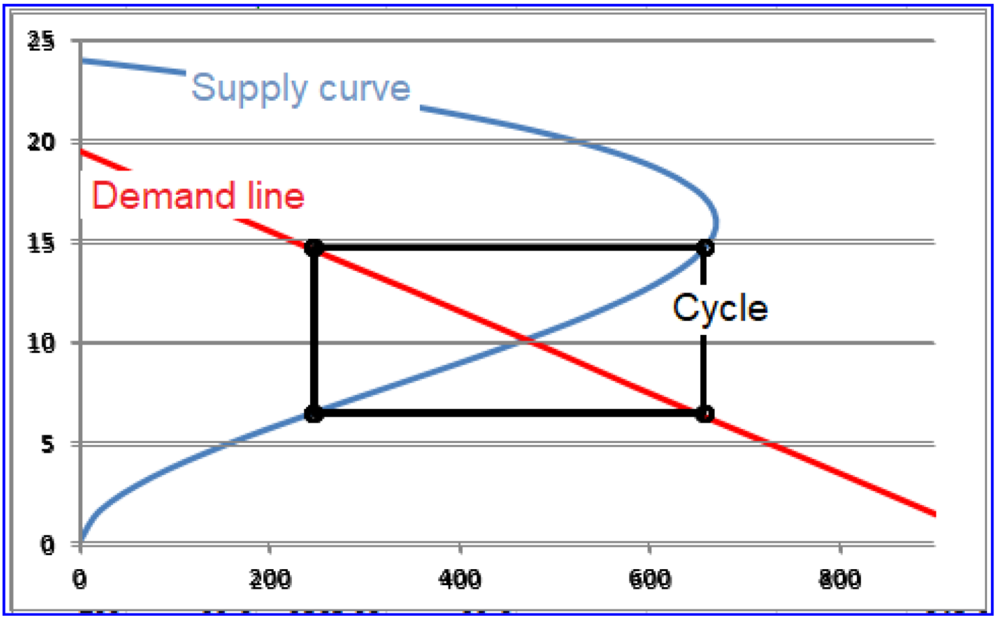
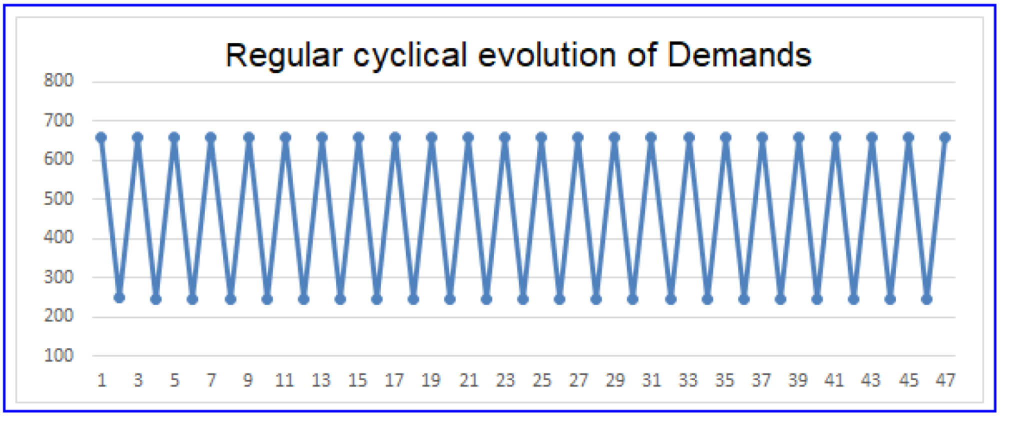
Figure 19: The emergence of a cycle in the virtual market: a slight backward bending of the supply curv



Source: author's calculations

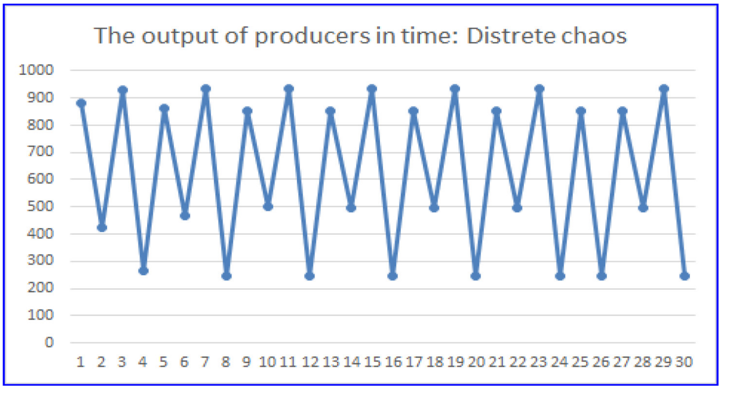
Figure 20: Evolution of the level of demand over time in Excel



Source: author's calculations

Based on our long-standing views on simulation models, we can state that virtual markets constructed in Excel can serve as a very good tool (toy) for learning about complex economic developments over time. We can also interpret this case as an ecological process. Behind the backward bending supply curve, we can identify different parts of the population of workers as owners with their labor forces diversified by their response to wage rates (prices). We identify the rest of the population as producers-employers. These agents set the first wage rate and, based on the response of employees, come up with the decision to start production using the offered quantity of labor. Let us experiment with the very simple case of the neoclassical production function $Q = a(K^{\alpha})L^{\beta}$, $a = 1$, $K = 100$ and is constant over time. It is logical to start with some low level of wage rates chosen by workers, let $L = 775$ in our Excel model.

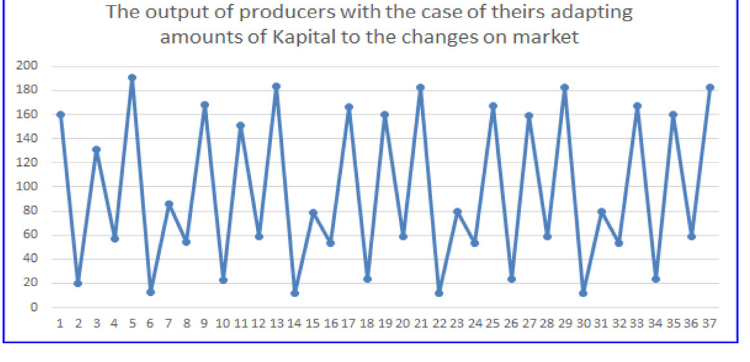
Figure 21: Saw plot of the output of our model in Excel: discrete chaos



Source: author's calculations

We can also try an experiment where workers respond to developments in the virtual market by changing the constant capital input to the production function.

Figure 22: Situation after running the model with capital level adjustment to market changes



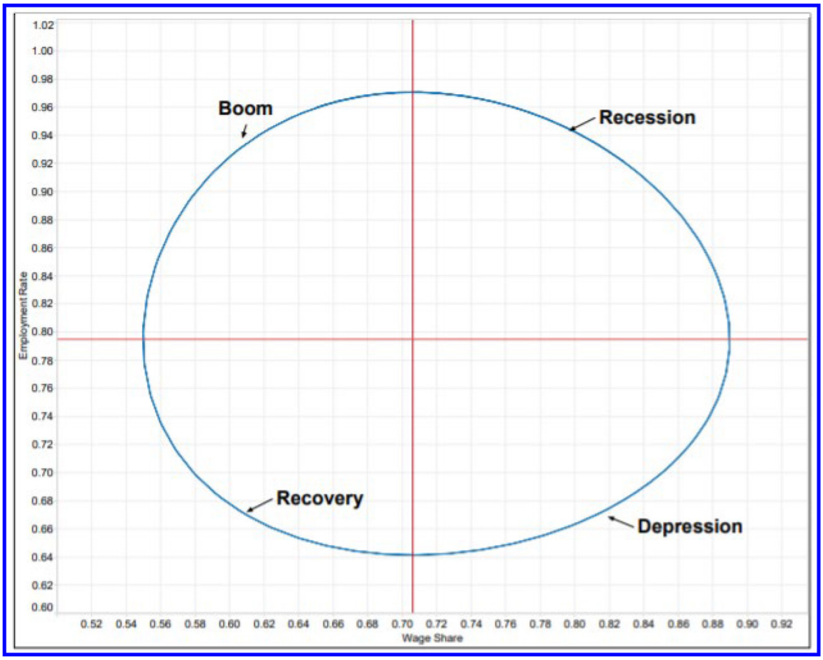
Source: author's calculations

EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 — 73
https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

4 Graphs of Goodwins cycles found in the world economic literature: observed real economies

Several essays can be found in the economic literature in which some authors attempt to perform tests on Goodwin's model set in objective economic reality. For example, researchers Grasselli and Maheshwari in their essay entitled "Testing Goodwin's Model with a General Capital Accumulation Rate" (2018) try to conduct econometric tests on a modified Goodwin's model in which the capital accumulation rate is constant but may not be equal to one as in the original model (Goodwin, 1967). In addition to this modification, they find that solving the methodological and reporting problems in Harvie (2000) leads to remarkably better results, with almost perfect agreement between estimates of equilibrium employment rates and the corresponding empirical averages, as well as significantly better estimates of equilibrium wage shares. Despite its simplicity and obvious limitations, the performance of the modified Goodwin model implied by their results shows that it can be used as a starting point for more sophisticated models of endogenous growth cycles.

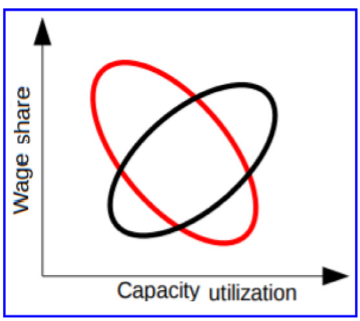
Figure 23: Solution with modified Goodwin model with parameter values $\alpha = 0.018, \beta = 0.02, \delta = 0.06, \gamma = 0.3, \rho = 0.4, v = 3, k = 1.$



Source: author's calculations

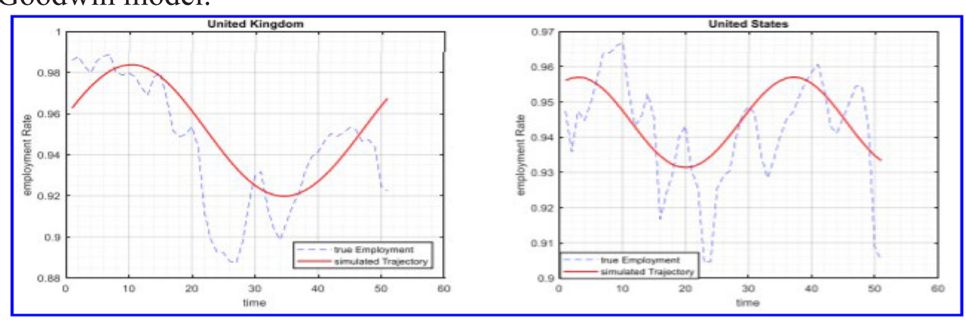
Also another scientist in this case, Tarassow (2010), attempts to verify Richard Goodwin's (1967) theory of the endogenous business cycle in his article *The empirical relevance of Goodwin's business cycle model for the US economy,* in which he argues that the driving force behind the fluctuations is the class struggle between capitalists and workers over the distribution of income. Based on the Marxist profit-based model, nonlinear differential equations lead to endogenous cycles in the wage-employment share space that can be empirically observed. Using a bivariate vector autoregressive model, Tarassow (2010) analyses the relationship between real labour unit costs and employment rates in the U.S. economy over the period 1948:1 to 2006:4. Granger causality tests, orthogonalised impulse response functions, and forecast error variance decomposition are performed for the raw data as well as the cyclical components using the Hodrick-Prescott and Baxter-King filter methods. Tarassow verifies the profitable nature of the U.S. goods market and finds that the income distribution is driven by labor market dynamics.

Figure 24: Wages as a share of capacity - Utilization cycle; Red orbit: profit-based regime, black orbit: wage-based regime



Source: Goodwin (1967, p.27)

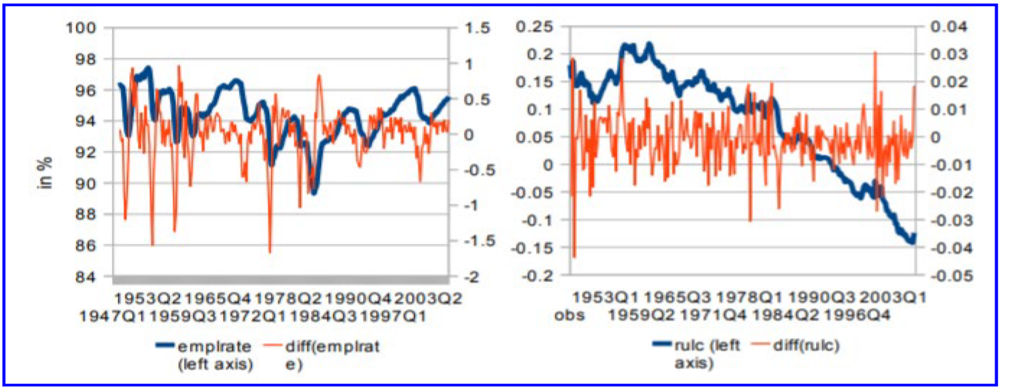
Figure 25: Observed and simulated employment rates for the modified Goodwin model.



Source: author’s calculations

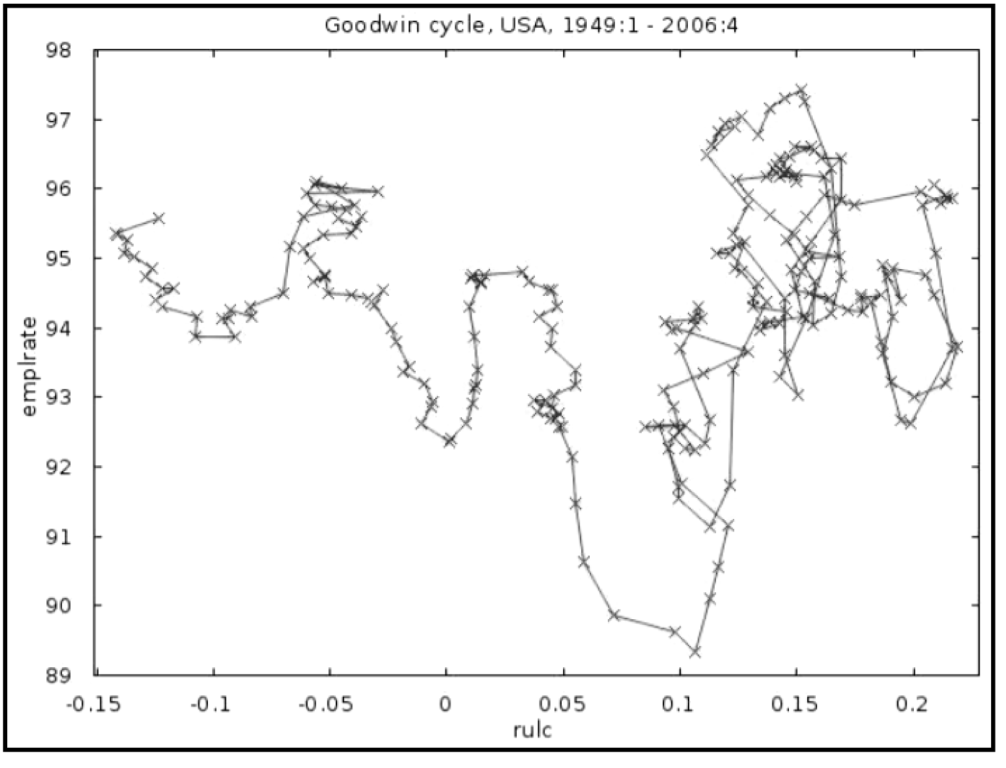
EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 — 73
https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

Figure 26: Overview of the time series used: emplrate - employment rate, rulc - log. Hourly real unit labour costs, USA, 1948:1-2006:4



Source: author's calculations

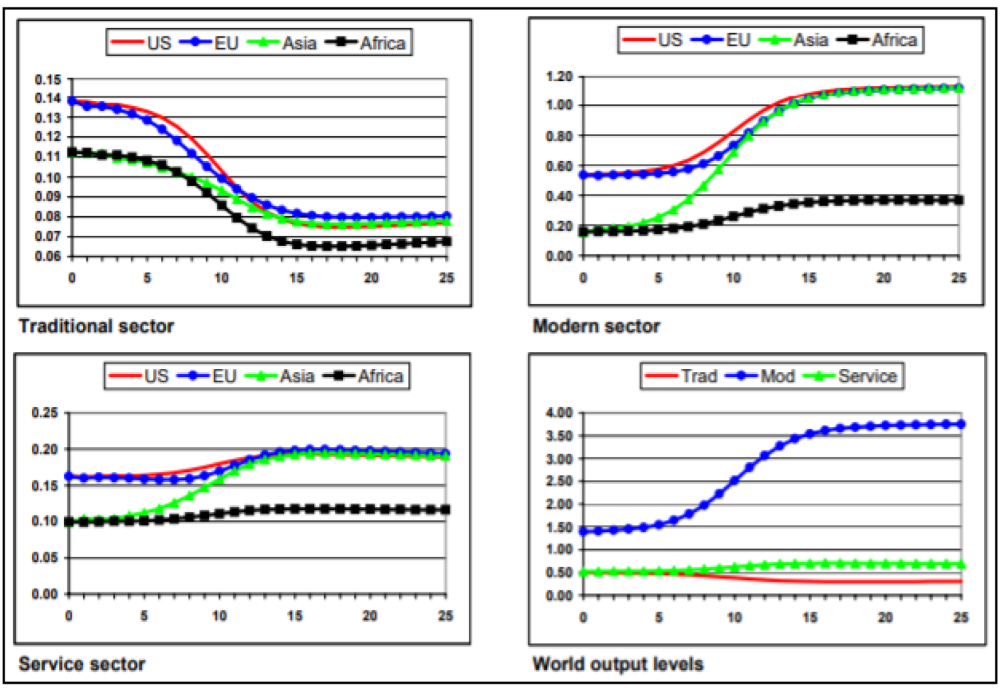
Figure 27: The Tarassow chart shows an overview of the interaction between employment rates and real unit costs from the interval 1948:1 to 2006:4



Source: Tarassow (2010)

Further, for example, among other scholars theorising on the basis of Goodwin's business cycle model, Landesmann and Stehrer (2006) write in the abstract of their article that they continue the work they started together with Richard M. Goodwin in the 1990s. Goodwin's later work went largely in the direction of modeling Schumpeter's insights about structural and technological transformations in the context of disaggregated models, while allowing for non-full employment outcomes and macroeconomic cyclical patterns to evolve alongside these transformations. In a series of papers, we have built on this work for closed and open economies, highlighting in particular the implications of structural transformations for macro distributional dynamics and effective demand issues. We analysed this for advanced and catching-up economies and their interdependencies on the global stage. In this context, we review our modelling efforts and trace them back to Goodwin's lifelong interest in synthesising disaggregated (linear) modelling with macrodynamic analysis. For comparison with the graphs we reproduced earlier, we have nudged some of the results implied by the authors in Fig. 29.

Figure 28: Output dynamics arrived at by two authors for different sectors in several economies

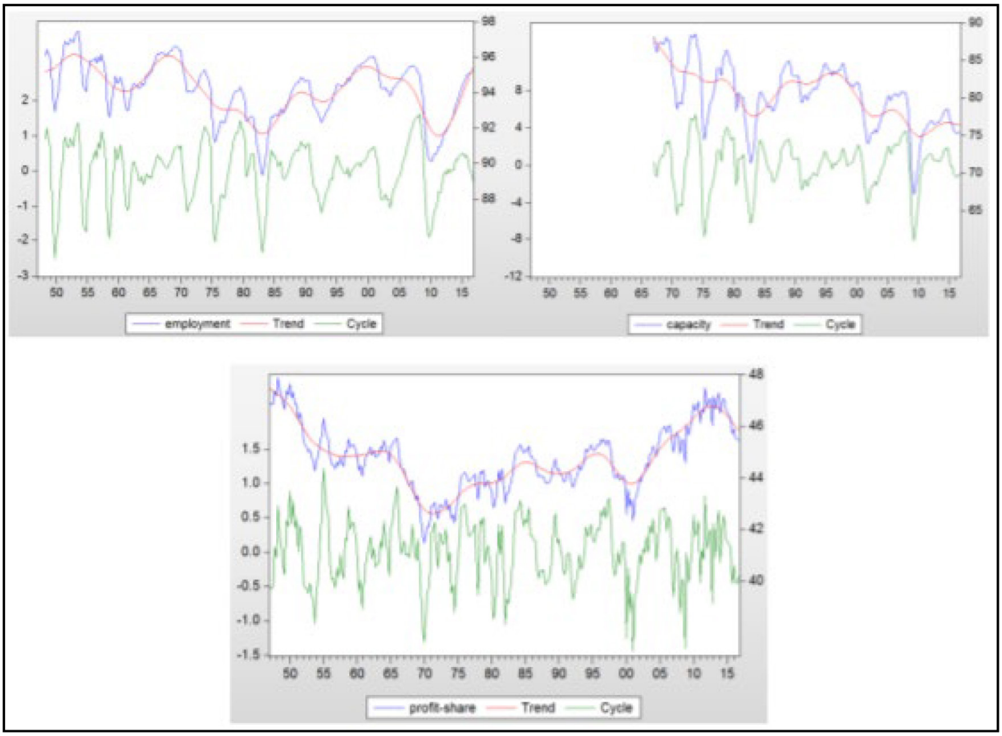


Source: [20] Landesmann, M., Stehrer, R., (2006)

EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 – 73
https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

Based on the observed statistical data and graphs published in the well-known world economic literature and in the works of the authors cited above, it is clear that the behaviour of economies in the long run is not smooth like a sine graph, but on the contrary, it is sawtooth.

Figure 29: Trend and cycle of employment, profit share and capacity utilisation

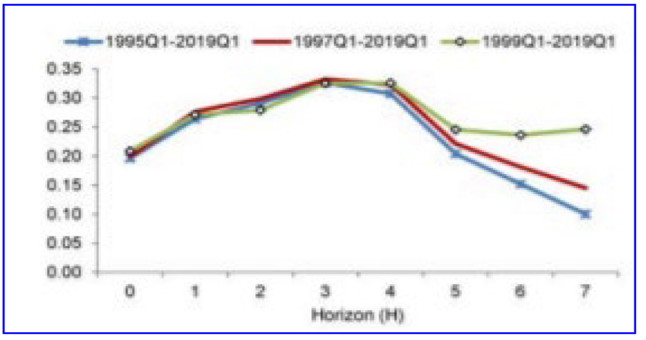


Source: Arajuo et al. (2017)

5 Graphs of economic development independently of Goodwins theory in several economies

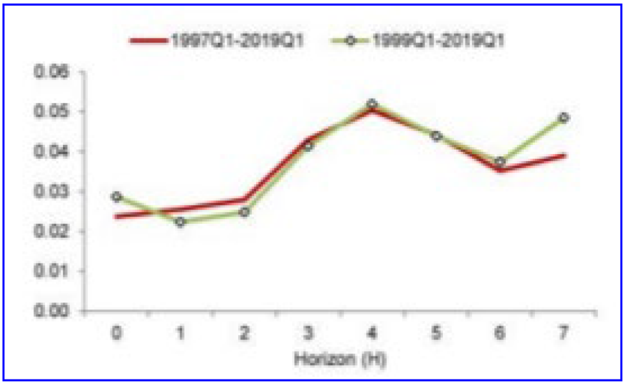
To confront the facts published above, we will present additional statistics and graphs of objective economic reality that demonstrate the fact that economies evolve in sawtooth trajectories, not as in conventional economies that evolve in smooth sinusoids.

Figure 30: Euro area linear ERPT: Import prices



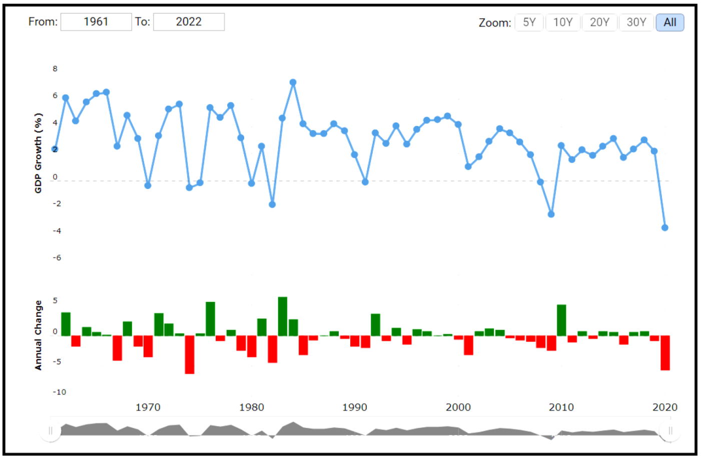
Source: Colavecchio & Rubene (2019).

Figure 31: Euro area linear ERPT: consumer prices



Source: Colavecchio & Rubene (2019).

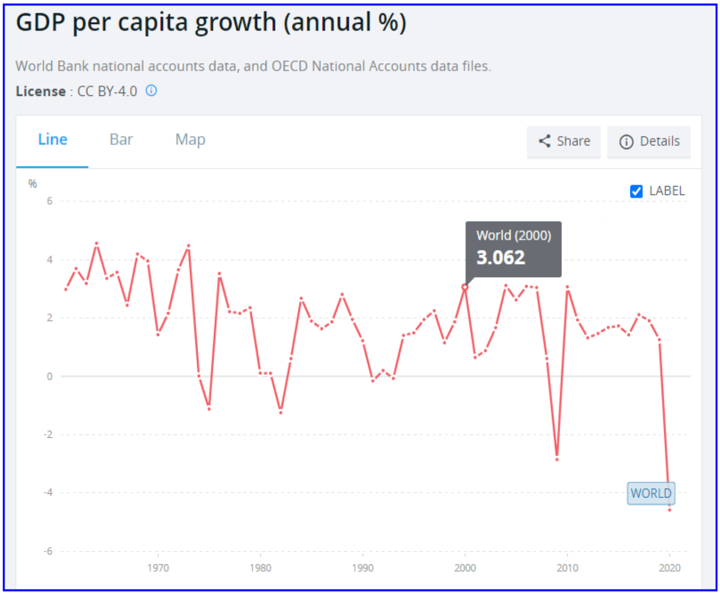
Figure 32: US GDP growth rate - a typical pie chart of the long-term evolution of the world economy (from the World Bank) Historical data from 1961 to 2022



Source: World Bank

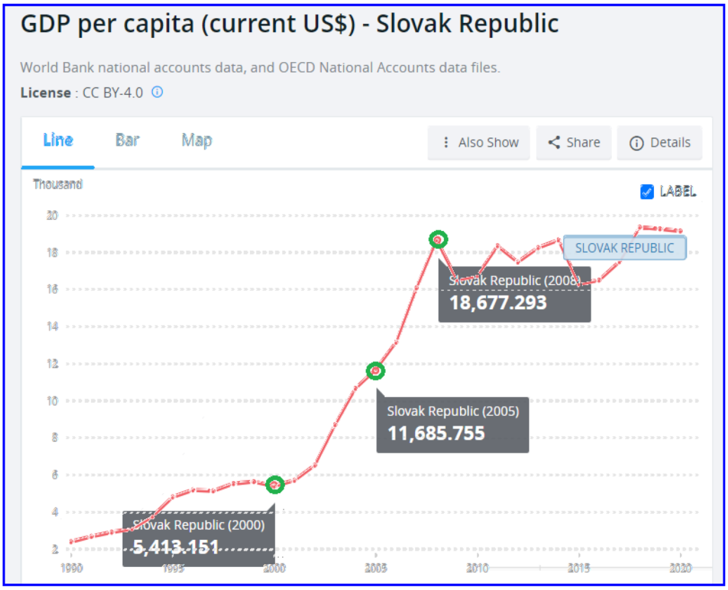
EKONOMICKÉ ROZHĽADY – ECONOMIC REVIEW, 2022, 51(1), 45 — 73
https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

Figure 33: Chart of the long-term evolution of the world economy (data from the World Bank) - GDP per capita growth



Source: World Bank, https://data.worldbank.org/indicator/NY.GDP.PCAP.CD?locations=SK

Figure 34: Chart of the long-term evolution of the Slovak economy (from the World Bank) - GDP per capita

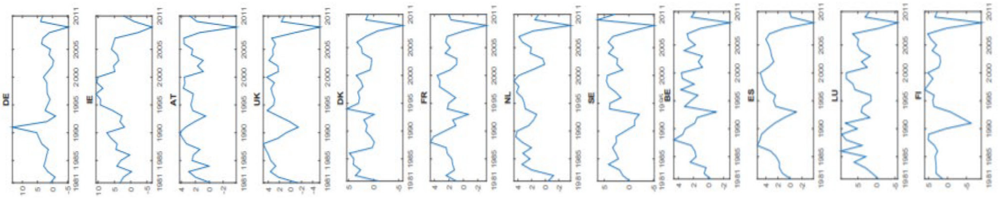


Source: World Bank, https://data.worldbank.org/indicator/NY.GDP.PCAP.CD?locations=SK

Figure 35: Chart of the long-term evolution of the Slovak economy (from the World Bank) - GDP per capita growth

Source: World Bank, https://data.worldbank.org/indicator/NY.GDP.PCAP.CD?locations=SK

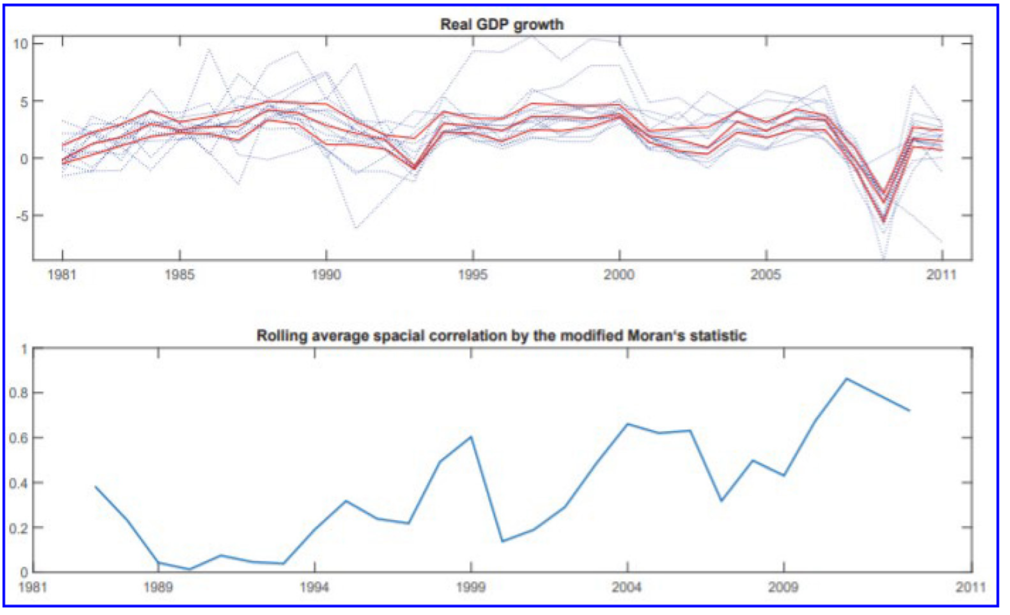
Figure 36: GDP growth rate. Several countries



Source: Bandrés et al. (2017)

https://doi.org/10.53465/ER.2644-7185.2022.1.45-73

Figure 37: Evolution and correlations between national growth rates

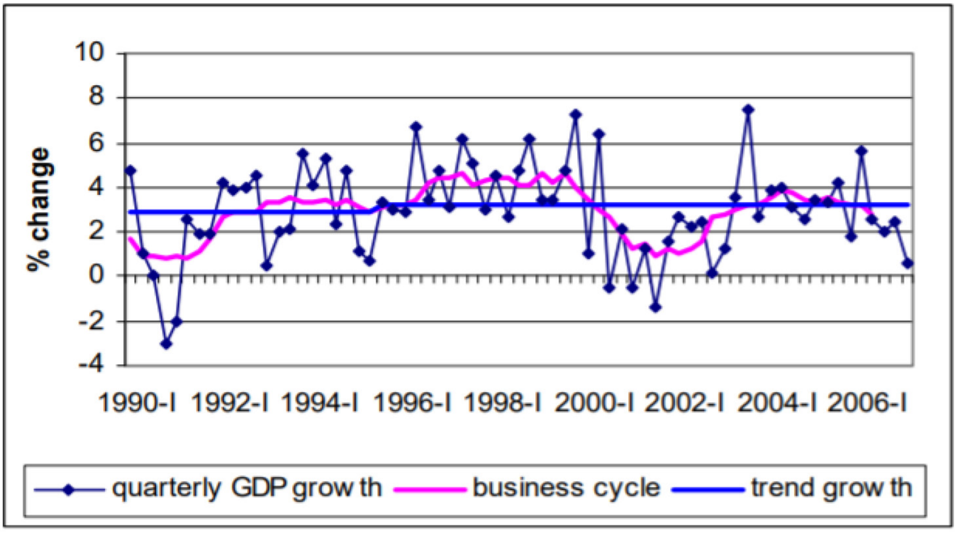


Source: Bandrés et al. (2017)

6 On some interesting approaches to non-linear economic growth as a comparison with ours

As the American economist Marc Labonte wrote in the summary of his report *Economic Growth and the Business Cycle* (2007) *"Business Cycles refers to the regular cyclical pattern of economic boom (expansion) and bust (recession). Recessions are characterized by a decline in output and employment; at the other end of the spectrum is the "overheating" of the economy, characterized by unsustainably rapid economic growth and rising inflation. Capital investment spending is the most cyclical component of economic output, while consumption is among the least cyclical. The government can mitigate booms and busts through monetary and fiscal policy. Monetary policy refers to changes in the Federal Reserve's overnight interest rate. When the Fed wants to stimulate economic activity, it lowers interest rates; when it wants to restrain economic activity, it raises rates. Fiscal policy refers to changes in the federal budget deficit. An increasing deficit stimulates economic activity, while a decreasing deficit restrains it. Policy changes designed to affect the business cycle inherently have only a temporary effect on the economy* *because booms and busts are transitory. In recent decades, expansions have lengthened and recessions have eased, perhaps due to better stabilization policies, or perhaps due to luck." On the other hand, it has been written that: ... " Long-term growth receives less attention from policymakers than cyclical growth. In the broader view of history, however, long-term growth is the more important of the two because it is the key to raising living standards. Long-term growth is driven by growth in labour, capital and productivity. Policy changes in education, taxation, competition, basic research and infrastructure can affect the long-term growth rate of an economy, but only marginally. Long-run growth has varied little throughout most of U.S. history, despite a wide range of policy changes. This fact is not so surprising when one considers that the main contributor to long-term growth is technological progress, over which the government has little direct influence. In recent years, long-run growth has accelerated slightly due to higher productivity growth, driven mainly by what is popularly referred to as the 'information technology revolution'. Although the government has had little direct influence on the IT revolution, it has created an environment in which these technological changes have been able to thrive, which probably explains why many other economies have not experienced similar productivity acceleration."* (our questions).

Figure 38: Quarterly GDP growth from 1990:I to 2007:I[3]



Source: Bureau of Economic Analysis, CRS calculations.

[3]Quarterly growth rates are annualized and seasonally adjusted. The business cycle data series is calculated using an eight-quarter moving average. The growth trend is calculated using an average with a turning point in 1995:3.

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7 Conclusions

We have presented a somewhat unconventional approach to understanding the features and causes of long-run nonlinearity of economic development as a whole. We take advantage of a promising ontological, methodological and theoretical perspective on the temporal complexity of evolving economies. In STELLA and Excel, we have run several Goodwin, Cournot and Ezekiel models.

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