BITCOIN VOLATILITY ANALYSIS: DETERMINISTIC AND PROBABILISTIC APPROACH

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Analýza volatility bitcoinu: deterministický a probabilistický prístup

Abstract: *Bitcoin is the phenomenon of the last few years and has dragged quite a lot of attention either from academicians or investors. Price fluctuations in the Bitcoin spot rate on the Bitcoin exchanges is driven by many factors; hence volatility analysis is of particular importance. In this paper we analyze deterministic approach represented by family GARCH (fGARCH) model and probabilistic approach by stochastic volatility model. Except for volatility time evolution, we involve short-term forecast. As a benchmark we use 30-days historical volatility. Based on mean average percentage error, family GARCH model tracks volatility better.*

Keywords: *Bitcoin, stochastic volatility, GARCH model*

JEL Classification: C73, G17

1. Introduction

We consider the financial market equilibrium with symmetric information with a single risky asset as presented by Jong and Rindi [8]. The representative risk-neutral agent receives an endowment of both the risky asset I and the risk-free asset I_f. At the end of the trading game there is payment from the risk-free asset $1 + r_f$ and from risky asset $\tilde{F}$ (normally distributed random variable).

The agent's objective function is:

$$\operatorname*{Max}_X E[u(\widetilde{w})], \qquad \widetilde{w} = (I + X)\tilde{F} + (I_f - Xp)(1 + r_f), \tag{1}$$

where u is agent's utility function, X is agent's demand function for risky asset and p is the price of risky asset. The price of risk-free asset is normalized to 1. The first order condition (f.o.c.) has the form:

$$\mathrm{E}\left[u'(\widetilde{w})\left(\tilde{F} - p(1 + r_f)\right)\right] = 0. \tag{2}$$

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Simplifying f.o.c. by using covariance formula[2] and Stein's lemma[3]we get:

$$\mathrm{E}[u'(\widetilde{w})]\mathrm{E}\left[\left(\tilde{F}-p\left(1+r_f\right)\right)\right]+\mathrm{E}[u''(\widetilde{w})](I+X)\mathrm{Var}\left(\tilde{F}\right)=0. \qquad (3)$$

From this equation, the equilibrium price follows immediately as:

$$p=\frac{1}{(1+r_f)}\left[\mathrm{E}[\tilde{F}]+\frac{\mathrm{E}[u''(\widetilde{w})]}{\mathrm{E}[u'(\widetilde{w})]}(I+X)\mathrm{Var}\left(\tilde{F}\right)\right]. \qquad (4)$$

In case of constant absolute risk aversion utility function $u(\widetilde{w})=-\exp(-A\widetilde{w})$where A is risk aversion constant, the price equation has the form:

$$p=\frac{1}{(1+r_f)}\left[\mathrm{E}[\tilde{F}]-A(I+X)\mathrm{Var}\left(\tilde{F}\right)\right]. \qquad (5)$$

Interpretation of (5) is quite simple: the price of the asset is its discounted expected cash flow less a discount for risk. The risk depends on:

1. the volatility of payoff,
2. the endowment of the risky asset,
3. the risk aversion.

The definition of volatility might differ depending on the point of view. Volatility usually refers to the variation observed in some phenomenon over time. Especially, statisticians and econometricians use volatility to describe the variability of the random component of time series. In our case, it is the volatility of return (payoff).

The amount of purchased assets is modifiable by decision makers as well as their attitude to the risk. On the other hand, the volatility evolves in time with only a small effect of some individual; therefore it is necessary to find a correct way to measure it. One approach is to model the evolution of volatility deterministically, i.e. through the (G)ARCH models. After the groundbreaking papers of Engle [4] and Bollerslev [1], these models have been generalized in numerous ways and applied to a vast amount of real-world problems. Taylor [11] proposed an alternative in his seminal work to model the volatility probabilistically, i.e. though a state-space model where the logarithm of the squared volatilities – the latent states – follow an autoregressive process of the first order. Over time, this specification became known as the stochastic volatility (SV) models which have found comparably

[2] Let x_1 and x_2 to be random variables, then:
$\mathrm{Cov}[x_1,x_2]=\mathrm{E}[x_1x_2]-\mathrm{E}[x_1]\mathrm{E}[x_2]$.

[3] Let x_1 and x_2 to be continuous, differentiable and jointly normally distributed random variables, then:
$\mathrm{Cov}[g(x_1),x_2]=\mathrm{E}[g'(x_1)]\mathrm{Cov}[x_1,x_2]$.

little use in applied work mostly due to the lack of standard software package implementation.

We have chosen to apply this symmetric game on bitcoin. Bitcoin is a decentralized digital currency without a central bank or administrator. During the last two years, bitcoin was able to reach the high approximately at \$19,345 and the low at \$2,559, which suggests strong fluctuation and ergo volatility analysis is worth to apply.

Being able to analyse historical development of volatility as well as to create accurate forecasts are crucial for risk management, portfolio selections and pricing financial instruments. The paper uses deterministic and probabilistic approach to Bitcoin volatility analysis. In the following section we present models and estimation methods, in the third chapter our empirical results are shown and the last chapter concludes.

2. Methodology

We describe fGARCH model in which all relevant information is observed and the model is correctly specified. The volatility is known, or predetermined, as of time $t-1$, and stochastic volatility models, if some relevant information is not observable, then it is possible to exploit a genuine subset of the full information set. Under this scenario, the true conditional variance will be unobservable, even under correct model specification and the volatility process becomes genuinely latent.

2.1 Stochastic volatility (SV) model

Let $\boldsymbol{y} = (y_1, y_2 \ldots, y_n)^T$ be a vector of returns with zero mean. Each return observation is assumed to have its intrinsic contemporaneous variance e^{h_t} therefore relaxing the homoskedasticity assumption. This variance does not vary unrestrictedly over time, it follows an autoregressive process of the first order. Following Kim et al. [10] SV centered parametrization is given:

$$y_t | h_t \sim \mathcal{N}(0, \exp h_t), \tag{6}$$

$$h_t | h_{t-1}, \mu, \phi, \sigma_\eta \sim \mathcal{N}\big(\mu + \phi(h_{t-1} - \mu), \sigma_\eta\big), \tag{7}$$

$$h_0 | \mu, \phi, \sigma_\eta \sim \mathcal{N}\big(\mu, \sigma_\eta / (1 - \phi^2)\big), \tag{8}$$

where μ is the level of log-variance, ϕ is the persistence of log variance and σ_η is the volatility of log-variance. We refer to $\boldsymbol{\theta} = \left(\mu, \phi, \sigma_\eta\right)^T$ as the vector of parameters. The process appearing in (6) - (8)$\boldsymbol{h} = (h_1, h_2 \ldots, h_n)^T$ is the latent time-varying volatility process (log-variance process).

The prior distribution of $\boldsymbol{\theta}$needs to be specified before setting the model. We choose independent components for each parameter, i.e. $p(\boldsymbol{\theta}) = p(\mu)p(\phi)p(\sigma_\eta)$. The level $\mu \in \mathbb{R}$ has the usual normal prior distribution

$\mu \sim \mathcal{N}(b_\mu, B_\mu)$, e.g. for daily log-returns $b_\mu = 0, B_\mu \geq 100$. A common strategy is to choose a vague prior; however, if one prefers to use slightly informative priors (to avoid outliers), one must pay attention to whether log-returns or percentage log-returns are analyzed. According to Kastner [9], exact choice is usually not very significant.

The persistence parameter ϕ is assumed to have beta distribution $(\phi + 1)/2 \sim \mathcal{B}(a_0, b_0)$. Clearly, the support of this distribution is the interval $(-1,1)$, thus stationarity is guaranteed. For financial datasets with not too many observations ($n \leq 1000$) the choice of hyperparameters a_0, b_0 can influence the shape of posterior distribution. Therefore, it may be useful to check the expected value and standard deviation of the persistence parameter given by Kastner [9]:

$$\mathrm{E}[\phi] = \frac{2a_0}{a_0 + b_0} - 1, \tag{9}$$

$$\mathrm{Var}(\phi) = \frac{4a_0 b_0}{(a_0 + b_0)^2 (a_0 + b_0 + 1)}. \tag{10}$$

It follows that expectation of the persistence parameter is positive if and only if $a_0 > b_0$, negative if and only if $a_0 < b_0$. In special case $a_0 = b_0 = 1$, the uniform distribution arises. The variance decreases with larger values of a_0, b_0.

Lastly, for the volatility $\sigma_\eta \in \mathbb{R}^+$ we choose $\sigma_\eta^2 \sim B_{\sigma_\eta} \times \chi_1^2 = \mathcal{G}\left(1/2\,, 1/2\,B_{\sigma_\eta}\right)$ which is motivated by Frühwirth–Schnatter and Wagner [5] who equivalently impose the prior of $\pm\sqrt{\sigma_\eta^2}$ to follow a centered normal distribution, i.e. $\pm\sqrt{\sigma_\eta^2} \sim \mathcal{N}\left(0, B_{\sigma_\eta}\right)$. The choice of hyperparameter B_{σ_η} turns out to be of minor influence as long as it is not set too small.

There are few methods to estimate parameters. The first method is based on Metropolis-Hastings algorithm. Early work on this method can be found in Hastings [6]. The algorithm can use different proposals of Inverse-Gamma function, such as the one presented above by Frühwirth-Schnatter and Wagner [5]. Conditional on a past draw of the volatilities, we loop through time, proposing new e^(h_t) and accept or reject them. Another method is based on the Kalman Filter. The AR(1)-SV model is a Gaussian non-linear state-space system. The method uses a Gaussian mixture approximation to the distribution of the error logarithm to turn the system into a linear, Gaussian one. One might also use a particle filter that uses an importance of sampling to sequentially construct weighted approximation to the sequence of posteriors.

2.2 GARCH type models

2.2.1 Conditional mean equation

The univariate GARCH specification allows to define dynamics for the conditional mean in terms of ARFIMAX model which may be formally defined as:

$$\Phi(L)(1-L)^d(y_t-\mu_t)=\Theta(L)\varepsilon_t \tag{11}$$

with the left-hand side denoting fractional AR specification and the right-hand side the MA specification. L is the lag operator, $d$is a fraction of long memory process and μ_t is defined as:

$$\mu_t=\mu+\sum_{i=1}^{m-n}\delta_i x_{i,t}+\sum_{i=m-n+1}^{m}\delta_i x_{i,t}\sigma_t+\xi\sigma_t^k, \tag{12}$$

where we allow m regressors $x_{i,t}$ of which n may optionally be multiplied by the conditional standard deviation σ_t. The term $\xi\sigma_t^k$ is referred as ARCH-in-mean if $k=1$ conditional standard deviation is involved, if $k=2$ conditional variance.

2.2.2 Conditional variance equation

Bollerslev (1986) presented standard GARCH model written as:

$$\sigma_t^2=\omega+\sum_{j=1}^{m}\xi_j v_{j,t}+\sum_{j=1}^{q}\alpha_j\varepsilon_{t-j}^2+\sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \tag{13}$$

where σ_t^2 is the conditional variance, $\omega,\xi_j,\alpha_j,\beta_j$ the parameters and ε_t^2 the residuals from the conditional mean. There are m external regressors $v_{j,t}$ upon the analyst's choice.

One of the key features of the observed behavior of financial data which GARCH models capture is volatility clustering which may be quantified in the persistence parameter:

$$\hat{P}=\sum_{j=1}^{q}\alpha_j+\sum_{j=1}^{p}\beta_j. \tag{14}$$

Related to this measure is the 'half-life'$(h2l)$ defined as the number of days it takes for half of the expected reversion back towards expected value of unconditional variance to occur:

$$h2l = \frac{-\ln 2}{\ln \hat{P}}. \tag{15}$$

In our analysis we use more complex model presented by Hentschel [7]. The family GARCH (fGARCH) model allows the decomposition of the residuals in the conditional variance equation to be driven by different factors for z_t (white noise process) and σ_t and also allowing shifts and rotations in the news impact curve – the shift is the main source of asymmetry for small shocks and rotation drives large shocks. Model is defined as:

$$\sigma_t^{\lambda} = \left(\omega + \sum_{j=1}^{m} \xi_j v_{jt}\right) + \sum_{j=1}^{q} \alpha_j \sigma_{t-j}^{\lambda} \left(\left|z_{t-j} - \eta_{2j}\right| - \eta_{1j}\left(z_{t-j} - \eta_{2j}\right)\right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\lambda} \tag{16}$$

which is a Box-Cox transformation for the conditional standard deviation whose shape is determined by λ, and the parameter δ transforms the absolute value function which it subject to rotations and shifts through η_{1j}, η_{2j}. Full fGARCH model presented by Hentschel implies $\lambda = \delta$. The persistence parameter:

$$\hat{P} = \sum_{j=1}^{p} \beta_j + \sum_{j=1}^{q} \alpha_j \kappa_j \tag{17}$$

where:

$$\kappa_j = E\left(\left|z_{t-j} - \eta_{2j}\right| - \eta_{1j}\left(z_{t-j} - \eta_{2j}\right)\right)^{\delta}. \tag{18}$$

Then the half-life numeric can be computed by formula (15).

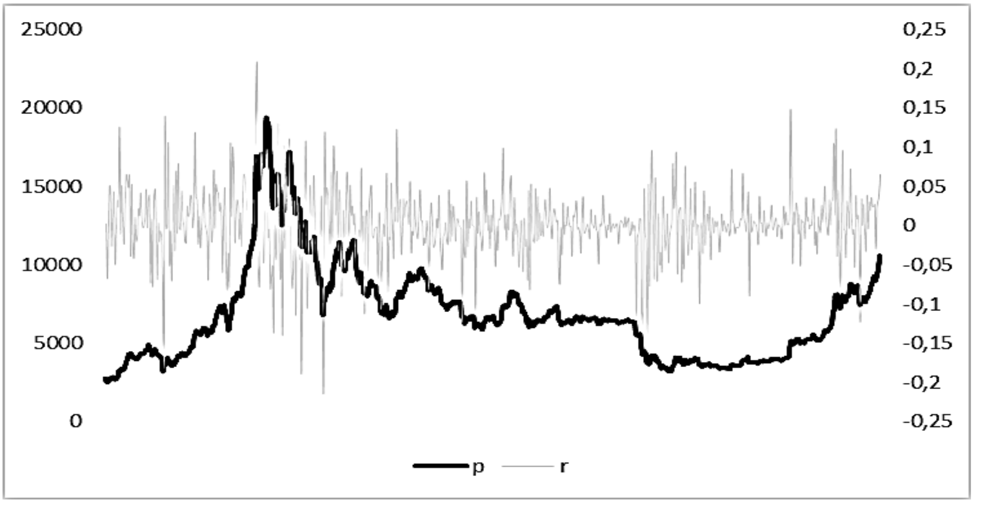
3. Empirical results

The Bitcoin analysis has received much attention mainly due to its simplicity, transparency and increasing popularity. Bitcoin is probably the most successful and controversial cryptocurrency, representing about 65% of the total estimated cryptocurrency capitalization [12].

The daily closing prices of Bitcoins during the period July 23rd2017 to June 22nd2019 were retrieved from www.stooq.com [13]. Our analysis is realized in RStudio. The evolution of prices and log-returns (natural logarithm of the ratio of two consecutive prices) are presented in Figure 1 and summary statistics in Table 1.

Figure 1

Time development of Bitcoin prices (p) and log-returns (r)



Source: author's calculations

Table 1

Summary statistics of Bitcoin prices (p) and log-returns (r)

Time series	Mean	Std. dev.	Ceof. var	Skewness	Kurtosis	ADF
p	6 854.26	3 033.19	0.4425	1.3490	5.2431	-1,7897
r	0.0019	0.0442	22.9659	-0.2609	6.0459	-25,7181***

Source: author's calculations

Notes: the variation coefficient (coef. var) is defined as the standard deviation divided by the mean and *** indicates the rejection of the null hypotheses at the 1% level

From Table 1 can be easily seen that average return is equal to 0.19 % with a standard deviation of 0.0442. The variation coefficient allows investors to determine how much volatility is assumed in comparison to the amount of return expected from investments. The lower value is, the better risk-return trade-off. According to the Augumented Dickey-Fuller (ADF) test we can reject null hypotheses in case of returns, hence, stationarity is guaranteed.

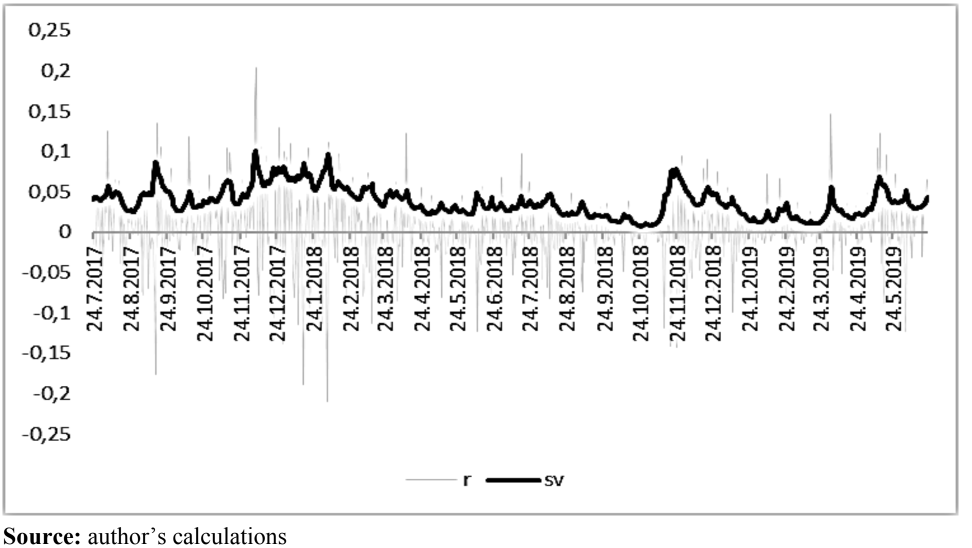
choice matters. We chose them so the expectation of persistence is relatively close to 1. Using Monte-Carlo Markov Chain estimation and presented prior distribution, we get following posterior moments:

$$\mathrm{E}[\mu] = -6{,}769, \qquad \mathrm{SD}(\mu) = 0.251;$$
$$\mathrm{E}[\phi] = 0.915, \qquad \mathrm{SD}(\phi) = 0.029;$$
$$\mathrm{E}[\sigma_\eta^2] = 0.242, \qquad \mathrm{SD}(\sigma_\eta^2) = 0.087.$$

The estimated volatility is presented in Figure 3 which represents probabilistic approach in defining the volatility, to be more precise, it is an empirical 50 % posterior quantile distribution of $100\exp\,(h_t/2)$ over time.

Figure 3

Stochastic volatility development



Source: author's calculations

In case of deterministic approach, we examine fGARCH model. In the first step, we define conditional mean equation in the form of ARFIMAX using autocorrelation, partial-autocorrelation functions and Ljung-Box Q test. The results for chosen lags are presented in Table 2, including Engle ARCH test.

Table 2

Statistics for conditional mean equation

	Q(1)	Q(200)	$Q^2(1)$	$Q^2(200)$	ARCH
r	0.4025	210.64	21.59***	403.06***	22.11***

Source: author's calculations

Notes: *** indicates the rejection of the null hypotheses at the 1 % level and Q^2 stands for the squared residuals

Based on the Ljung-Box Q test, we use level conditional mean equation. We modify fGARCH and parameters are estimated assuming generalized error distribution. The form of the estimated model is:

$$\mu_t = 0.0011,$$

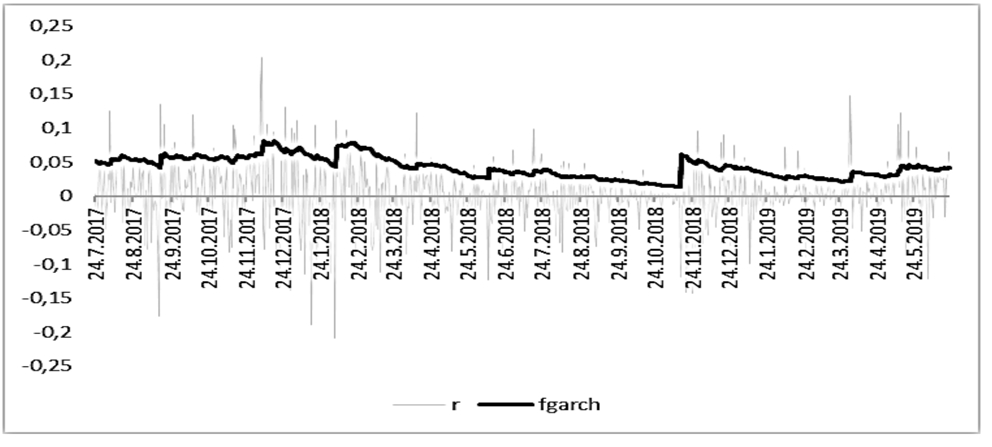
$$\sigma_t^{2.49} = 0.126\sigma_{t-1}^{2.49}\left(|z_{t-1} + 2.931| - 0.691(z_{t-1} + 2.9309)\right)^{2.49} + 0.873\sigma_{t-1}^{2.49}.$$

All estimated parameters are statistically significant. The persistence parameter has the value 0.999 which corresponds to our assumption in SV model. The volatility development is presented in Figure 4.

For completeness we use a thirty-day historical volatility as a benchmark, the time evolution is presented in Figure 5. For easier comparison, we involve some basic statistics in Table 3.

Figure 4

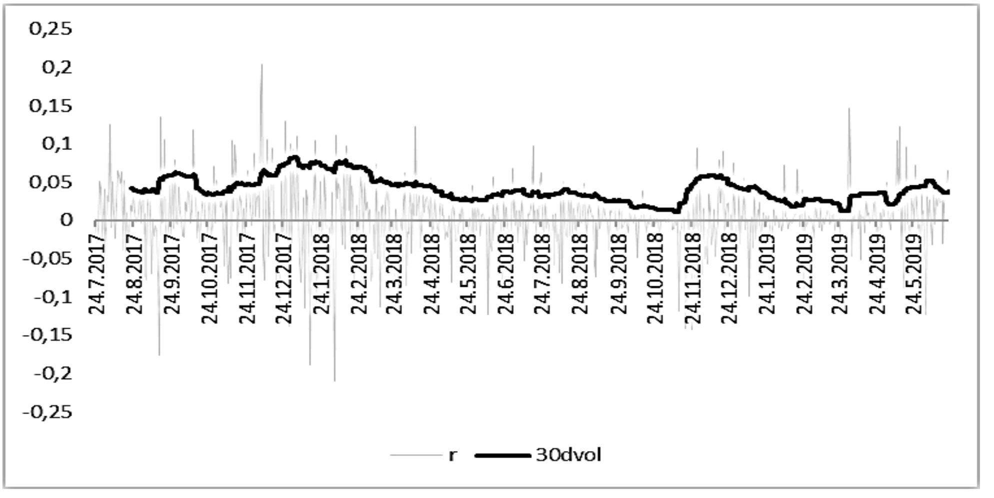
fGARCH volatility development



Source: author's calculations

Figure 5

30-days historical volatility



Source: author's calculations

Table 3

Summary statistics of volatilities

	Mean	Median	Min	Max	Std.dev.	Coef. var
sv	0.0376	0.0346	0.0074	0.1008	0.0178	0.4716
fgarch	0.0424	0.0409	0.0130	0.0805	0.0155	0.3650
30dvol	0.0409	0.0378	0.0107	0.0829	0.0162	0.3948

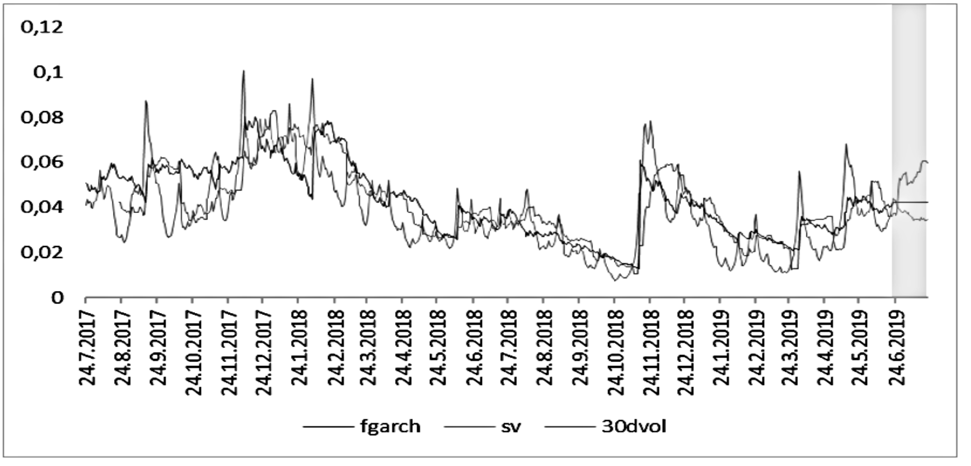
Source: author's calculations

If we use 30-days volatility as a benchmark, both models have tendency to underestimate risk. If we use different quantile in SV model (for example 95%), it would lead to a different conclusion.

Using either way, the highest point was found in all models the same in December 2017. They also pointed out the similar decreasing and increasing tendencies. For better assessment we also include 30-days forecast in Figure 6.

Figure 6

30-day volatility forecast



Source: author's calculations

Based on mean absolute percentage error (MAPE), fGARCH model seems to be better matching historical volatility. Deterministic approach appears to be handling even forecasting somehow better. MAPE results are presented in Table 4 (30-days historical volatility is used as a benchmark).

Table 4

Summary statistics of volatilities

MAPE	historical	forecast
sv	0.2656	0.3069
fgarch	0.1727	0.2124

Source: author's calculations

4. Conclusion

Bitcoin is primarily used for investment purposes; hence volatility analysis is of high importance. This paper investigated the ability of probabilistic and deterministic modelling. We found evidence that optimal model in terms of MAPE is fGARCH, although, stochastic approach allows different quantiles and different autoregression scheme, then different conclusions might come up.

Bitcoin is an asset which creates possibilities for stakeholders with regards to risk diversion and portfolio analysis. Hence, it can be a useful to investors to make more informed decision.

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References

[1] BOLLERSLEV, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. In *Journal of Econometrics*. Vol. 31, Issue 3, pp. 307 –327. ISSN 0304-4076.

[2] CULLEN, A. – FREY, H. 1999. *Probabilistic Techniques in Exposure Assessment.* New York: Springer, 1999. ISBN 978-03-06459-566.

[3] EFRON, B. – TIBSHIRANI, R. 1994. *An Introduction to the Bootstrap.* New York: Chapman & Hall, 1994. ISBN 978-0-412-04321-7.

[4] ENGLE, R. F. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. In *Econometrica.* Vol. 50, Issue 4, pp. 987– 1007. ISSN 1468-0262.

[5] FRÜHWIRTH-SCHNATTER, S. –WAGNER, H. 2010. Stochastic Model Specification Search for Gaussian and partial Non-Gaussian State Space Models. *Journal of Econometrics*. Vol. 154, Issue 1, pp. 85– 100. ISSN 0304-4076.

[6] HASTINGS, W. K. 1970. Monte-Carlo Sampling Methods Using Markov Chains and Their Applications. *Biometrika*. Vol. 57, Issue 1, pp. 97– 109. ISSN 0006-3444.

[7] HENTSCHEL, L. 1995. All in the Family: Nesting Symmetric an Asymmetric GARCH models. *Journal of Financial Economics*. Vol. 39, Issue 1, pp. 71–104. ISSN 1879-2774.

[8] JONG, F. – RINDI, B. 2009. *The Microstructure of Financial Markets*. Cambridge: CUP, 2009. ISBN 978-0-521-86784-9.

[9] KASTNER, G. 2016. Dealing with Stochastic Volatility in Time Series Using the R Package stochvol. *Journal of Statistical Software*. Vol. 69, Issue 5. ISSN 1548-7660.

[10] KIM, S. – SHEPHARD, N. – CHIB, S. 1998. Stochastic Volatility: Likelihood Inference and Comparision With ARCH Models. *Review of Economic Studies*. Vol. 65, Issue 3, pp. 361-393. ISSN 0034-6527.

[11] TAYLOR, S. J. 1982. Financial Returns Modelled by the Product of Two

Stochastic Processes: A Study of Daily Sugar Prices. In Anderson, T.W. et al. (eds.) *Time Series Analysis: Theory and Practice 1.* Amsterdam: North-Holland, pp. 203– 226. ISBN 978-04-44863-379.

[12] www.coinmarketcap.com [accessed 2019-08-01].

[13] www.stooq.com [accessed 2019-08-01].